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MEGATECTONICS AND GEOPHYSICS

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Abstract--By means of specific examples, a graphic picture is presented of the growth of geologic thinking, from the mapping of local geology to the nature of the visible part of the Earth's crust as a whole. The development of three basic concepts is briefly sketched which make possible, significant integration of areal geological knowledge. They are: (1) geosynclines, as loci of orogenic deformation; (2) ultramafics as initial products of orogeny; and (3) metamorphism and granite as products of mature orogeny. The synthesis of the relations implied in these concepts leads to a rather unorthodox outlook on fundamental questions concerning the history and dynamics of the Earth's crust which concern geophysicists as much as geologists. It looks as if there had never been a primeval upper, sialic part of the Earth's crust, but rather as if the sial was a by-product of orogenic deformation. The question seems not to be "Why is there no granite on the (main portions of) the ocean floors?" but "Why is there granite in the continents?" Correspondingly, the big question is not "Why are the ocean floors low?" but "Why are the continents high?" The answer to these questions lies fundamentally in the nature of orogeny.

Measured by their objectives and methods, geophysics and geology have differed radically from their beginnings and have developed in opposite directions. Eratosthenes set the pace for geophysics when, some two hundred years before Christ, he measured the angular difference between two towns in Egypt located approximately on the same meridian and, using the army engineers' figure for the surface distance between them, made the first attempt to compute the circumference of the Earth. Thus, geophysics, growing out of astronomy, geodesy, and navigation, had as its chief aim the comprehension of the whole, be it the body of the Earth, or the oceans or the atmosphere, or the gravity and magnetic fields surrounding them.

The basic approach to its problems was therefore mathematical, the attempt to derive first approximations to the overall picture from equations that draw a maximum of knowledge from relatively few data. Gradually, especially in the last half century, the theoretical and practical demands for growing refinement have led the geophysicist to dwell more and more on the disturbing complications of local detail. Thus, studies in microclimates have appeared within the larger frame of climates; oceanographers are deciphering the multitudinous deviations of currents within the broad sweep of oceanic circulation; the loose network of gravity and magnetic observations is being replaced by precise, large-scale maps of gravity and magnetic anomalies; and seismologists make their own local quakes to trace the elastic waves between closely spaced stations on land or between ships at sea. Everywhere the trend has been from the generalized to the specific, from the body of the Earth to the details near its surface; and here the aims of geologists and geophysicists meet.

By its very nature, geology had to develop in the opposite direction. Geological knowledge concerns the nature, structure, and development in time of the rock materials of an area. Because of man's microbic size compared to the surface of the Earth, the area he can cover with sufficient care is always very small compared to that of a continent, not to speak of the Earth as a whole.

Since the geologist deals with a multiplicity of materials, bearing the most diverse relations to those of surrounding points not only in space, but above all also in time, the power of the mathematical tools is denied him. He must operate primarily with generalized qualitative concepts, derived from the analysis of local conditions.

Gradually, the local knowledge accumulating fortuitously in different parts of a country was integrated, like the pieces of a gigantic, badly damaged, and incomplete jigsaw puzzle; geological county maps were united into larger units, eventually into maps of whole countries, each step leading to a broadening and deepening of general concepts. And thus, slowly, the creative thinking of geologists was led from the problems of their own little corners to those of continents and to the

Earth's crust as a whole; and from the concrete observations of the individual field work to the more abstract task of integrating regional knowledge and creating valid general concepts in terms of which the realities of the Earth's crust can be expressed adequately and submitted to the geophysicist and geochemist for interpretation in terms of basic principles.

Speaking as a geologist before geophysicists of all persuasions, I have chosen to present a graphic picture of this growth of geological thinking from the local detail to the nature of the visible part of the Earth's crust as a whole. We shall trace in barest outline the development of three basic concepts and the effect of the integration of areal knowledge concerning them on the problems of crustal deformation that are common to geology and geophysics.

Geosynclines as loci of orogenic deformation

I shall begin, where all creative thinking in geology must begin, with the geology of a few square miles of land, in this instance in the vicinity of the Hauenstein tunnel in the Swiss Jura Mountains, as shown on an instructive plate published by THORNBURG [1926].[a] The main section is drawn on the scale 1:2000, that is, one inch equaling 166 ft. It shows the actual exposures seen on the hillside, in the creekbed, the tunnel, an air shaft, and an inclined drainage tube. On the original map, the location and extent of all exposures are marked and the attitude of the rock layers is indicated as to azimuth and dip. The relative age of the beds, which in all such cases is established by remarkably complex lines of reasoning proven valid by experience, is indicated by colors.

Many such large-scale field maps are reduced in scale and generalized into a regional map, like that shown in Figure 1 of Thornburg's plate which is drawn on the scale 1:25000 or one inch equaling about 0.4 mi. Here a symbol is used to record the presence of the inclined fractures along which the beds were broken and piled one on top of the other. But these thrust faults are merely modifications of the folds which dominate the map picture. The folds seen in this small area are part of a regional fold pattern. To comprehend the nature of this pattern, many such maps must be combined on a still smaller scale and the confusing bands of individual rock formations must be replaced by a line symbol representing the fold axis. In HEIM'S [1919, Fig. 103, p. 615] classical diagrammatic map, the fold axes are shown as lines with a width proportional to the height of the fold. Thus, comprehension of the essential aspects of regional structure begins with the formulation of basic concepts, such as fold axes, thrust planes, and tear faults, and their representation by symbols on maps of smaller scale. Heim's graphic picture of the prototype of folded mountains covers approximately 7500 sq mi. It represents part of the essence of geological field studies extending through 150 years. DE MARGERIE'S [1922] great annotated bibliography of the geological literature of the Swiss-French Jura folds is a quarto volume of 300-odd pages. Yet these folds occupy only about 1/20,000 of the Earth's land area and 1/68,000 of the surface of the Earth.

The Jura folds that look so much like wrinkles produced in a tablecloth by the push of a hand, upon closer geometrical analysis prove to be actually only the surficial sloughing off of the outer sedimentary skin, a relatively insignificant marginal effect produced by the more fundamental deformation recorded in the Alps [BUXTORF, 1907]. There, however, as in the central parts of other great folded mountain belts, the structural relations are so complex that concepts more penetrating that those which mere geometrical analysis can yield are needed, if the local structure is to be tied into a comprehensible globe-encircling picture.

The first of the three basic, unifying concepts to be here discussed arose in the United States, in the mind of a stratigraphic paleontologist. In 1857, James Hall announced that he had observed that as one goes from the undeformed plateau regions of western New York, Pennsylvania, West Virginia, and Kentucky, eastward toward the Appalachian folded belt, sediments of the same geological age thicken progressively. This means that the surface of the basement complex on which the Paleozoic sediments of the region were deposited must be bent down to great depths. The resulting elongated regional depression he called "geosyncline."

Today, large numbers of detailed measurements of stratigraphic thicknesses, checked by thousands of drill holes, all made originally for local purposes, have been combined to form a graphic picture of the western half of this ancient Appalachian geosyncline. The base of the marine

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[a] In this paper, maps and other illustrative material were shown as slides during the lecture, but are omitted from the published version. They can be consulted in the original publications, which are all listed under "References" at the end of the address.

Silurian beds lies, for instance, progressively farther below the top of the Pennsylvanian Pottsville formation, as is shown, for example, by the isopach lines in a map published by PRICE [1931]. Extrapolating beyond the interval covered by this and similar maps, MOSS [1936, pl. 2] has attempted to show the conservatively estimated depth at which the Pre-Cambrian basement lies today in the western part of the central Appalachian folded belt. The greatest depth of 20,000 ft below sea level is probably 50 per cent or more too small.

Compare this three-dimensional picture of the geosynclinal trough with the tectonic map of the same region [KING and Others, 1944]. On this map, as in the more detailed map of the folded Jura, major fold axes and thrust planes are shown only in the marginal, here western, part. The structure of the eastern half, where the real core of the deformation lay, here as in the Alps, defies as yet adequate symbolic map representation.

The comparison of isopach maps with tectonic maps brings out clearly the importance of Hall's generalization. The clue to the fundamental nature of the deformation lies in the recognition of the general truth that an elongated depression, a geosyncline, forms first where later the crust is crumpled into a great belt of folded mountains.

Please note that I am presenting these samples of structural synthesis in order to illustrate graphically the main theme of my address, the systematic growth of demonstrable geological knowledge from the specific and local to general, fundamental insight. In order to show how new regional knowledge is constantly bearing out the generality of the concept of geosyncline and its causal relation to the loci of mountain folding, we turn to the world that today lies behind the iron curtain. The map shows the thickness of the sediments of one geologic period only, the Upper Carboniferous (or Pennsylvanian) between the longitudes of Odessa and Tobolsk [VON ZUR MÜHLEN, 1940, Fig. 11, p. 631]. In the lowest part, the thickness approaches 4000 ft for one geologic period alone. Note the two troughs, each of which was later compressed into a folded belt, that of the Donetz basin in the southwest, and that of the Urals in the east. No tectonic map, comparable to that of the United States, is as yet available for the Urals. But the (generalized) geological map [SAFONOV, 1937, map on pp. 1448-1449] suggests again the contrasts between simpler marginal folds on the west side and a complex eastern core.

For a still broader view of the distribution of folded geosynclines, seen on maps of still smaller scale, the two criteria (unusual thickness of sedimentary series and compression into folds), must be used to bridge the wide gaps that exist even in generalized reconnaissance knowledge. On ARKHANGELSKY'S [1937, pl. 10, colored map, "Tectonics of Eurasia"] structure map of Eurasia, single trend lines stand for whole belts of folds and the existance of geosynclinal thicknesses is indicated by distinctive color tones. Here the folded Ural appears only as part of a wider belt within which geosynclinal troughs came into being successively, later ones lying beyond and roughly parallel with their predecessors. Here, as elsewhere, this statement is true only in the most generalized sense and omits a multitude of diversities and complexities; and yet, in spite of its broad generality, the concept of this growth of a continental mass from successive belts of folding, is of fundamental importance, because for any specified fraction of geological time, these belts form a pattern. They converge toward south central Asia from the west, the north, the northeast, and the southeast.

In a generalized way, this alignment is reflected in HAUG'S [1900; redrawn in Bucher, 1933] picture of the distribution of the belts of geosynclinal sediments of the last 200 million years, that is, since late Paleozoic times. This map picture is the last stage of abstraction and generalization, so far removed from concrete reality that it serves only one purpose: to bring out the fact that geosynclinal down-buckling and failure did not take place wholly at random on the face of the Earth, but according to a recognizable, though but vaguely distinguishable pattern. It reflects the great asymmetry in the face of the Earth: on one side of the globe, a great knot of highlands where belts of earlier geosynclinal sinking and later orogenic crumpling crowd together in west central Asia, whence they diverge in such a manner as to surround a vast uncrumpled area on the opposite side, the Pacific basin.

Ultramafic intrusives as initial products of orogeny

The generalized picture of individual orogenic belts, especially the youngest and therefore still highest folded mountain ranges was extended into depth by the geophysicist, especially through gravimetric and seismic studies. Vening Meinesz' classical studies of gravity in the East Indies led to the recognition of belts of abnormally high negative gravity value that roughly coincide with the youngest and now active orogenic belts. The analysis of the records of earthquakes that have crossed one of the young folded mountains, such as the Sierra Nevada or the Eastern Alps, has proven the presence of lower-speed, that is, lower density, materials down to depths below the surface of as much as 40 km. Thus, the crumpling at the surface, which has produced the folded mountain ranges, appears as only the smaller part of a process that affects the whole crust.

Through the analysis of the gravity anomalies, VENING MEINESZ [1934, Fig. 19, p. 119] was led to his familiar hypothetical concept. This figure shows his conservative picture of a mountain root, formed of the lighter part of the crust extending down into the lower, heavier part, embedded in it like the roots of a tooth in the jaw.

This is the concept from which all reasoning concerning causes of crustal deformation starts. Somehow, the material of the Earth's crust is being crowded together. Geophysicists are inclined to look toward subcrustal convection currents, involving the slow, solid flow of subcrustal materials as the cause of such local crowding, the thinness of the crust and its relative weakness suggesting a passive role. As long as our thinking is based only on a two-dimensional cross-section, the picture of convection currents is satisfactory especially if we contemplate the section across the west border of the Pacific where there is a strong contrast between the basalt of the ocean floor and the lighter rocks of the continent, which vaguely suggests a possible reason for convection at depth.

But the assumption of convection currents appears less plausible the closer we look at the third dimension. Consider the great belt of young folded mountains that skirts the eastern border of Asia, swings through a hairpin curve around the Banda Sea to Sumatra and on into Burma. There it doubles back and runs through the Himalayas and the ranges of Baluchistan, Iran, and Asia Minor into the narrow space between Africa and Europe. For about half of this distance it separates ocean and continent; for the other half, it lies between old continental masses, with most of Eurasia on one side and with the Indian Peninsula, Arabia, and Africa on the other. The assumption of minor convection currents superimposed on the major pattern reminds one of the cycles and epicycles by which Ptolemaic astronomy explained the motions of the planets. Seen in the perspective of the whole orogenic picture, the concept of subcrustal convection as the reason for orogenic belts is as little convincing today as when it was first set forth brilliantly by David Griggs.

There may be an alternative. In a paper presented before the 14th International Congress in Madrid, in 1926, QUIRKE [1928] reported on crude experiments. He covered a big rubber balloon with a mixture of vaseline and paraffin, placed it in a hydraulic tank and slowly let the air out. The crowding together of the vaseline was taken up along one or two great circles. For a model experiment that reduces the 12.7 million meters of the Earth's diameter to a meter or less, and 1.8×10^{11} days (500 million yr) since the beginning of the Cambrian to a small fraction of a day, the vaseline mixture used may have come near being dimensionally correct.

Two years earlier, I reported before the Geological Society of America, experiments with sponge rubber balls covered with a thin coating of paraffin and subjected to compression in a hydraulic tank. In one case, when the water surrounding the sphere was kept rather warm, deformation began at one point, producing three oblique thrust planes over which the soft paraffin curled in three wrinkles that diverged from a common point [BUCHER, 1924, Fig. 16, p. 288]. At the time, I did not appreciate the implications of this result. It is probable that had I continued, the inhomogeneities of the shell would have deflected the shear fractures so that, instead of meeting in a point on the antipodes, the lines of fracture would have bent around, encircling an irregular calotte. (This is what happened when similar shells were fractured under tension produced by expansion of the subcrustal volume.)

It seems possible, therefore, to assume that the difference between the land and water hemisphere is the difference between the side on which a spherical shell fails first along cracks that radiate out in several directions, and the opposite side, a good part of which tends to remain undeformed. But now it seems to me that while Quirke's tests show, in a general way, that a thin, very weak outer shell surrounding a shrinking sphere tends to crumple along a few globe-encircling wrinkles, my test shows more specifically that, with somewhat greater strength, bending and shearing will start at one point on the shell and produce along diverging lines the type of polarity that is the most striking property of the pattern of the Earth's orogenic belts.

This alternative is unattractive to the geophysicist because the deflation of the balloon or compression of the rubber ball suggests an overall volume change of the Earth. But on a body with an acceleration of gravity at the surface such as that on Earth, the same effect would be achieved in the crust if the reduction in volume were confined to a relatively thin shell just below the crust.

Throughout the Earth's history, there has been a transfer of matter in the liquid and gaseous state from below to the surface. There is growing evidence that even more matter has been introduced into the crust than through it. If in that way the volume of the crust should become enlarged and that of the immediate subcrust reduced, large effects might result. All evidence of transfer of matter within the crust becomes, therefore, of fundamental importance to all thinking about the deformation of the crust. This takes us to the second basic concept with which this paper is concerned.

In 1911, Dewey and Flett pointed out that the basic lavas that are found commonly associated with sediments accumulating in a geosyncline tend to differ from otherwise similar lavas that reach the surface outside the geosynclines. They are generally much richer in sodium (held chiefly in the mineral albite), and in water and carbon dioxide (held in such minerals as chlorite, epidote, and calcite). Following tradition, the appearance of these distinctive "spilitic" lavas, as they were called, was ascribed to the presence beneath the crust of an unique potential melt or magma, ready to rise to the surface as lava when given a chance. But since in the course of geologic history new geosynclines have appeared beyond the position of the earlier ones, this coincidence appeared strange indeed, unless the chemical nature of the lava had something to do with the process of geosynclinal sinking.

But volcanism tends to yield such heterogeneous products that little of general validity can be expected from studies of surficial lavas. Something more significant came to light when studies, such as by W. N. Benson in New South Wales and N. Sundius in Sweden, made clear that in connection with the first crumpling and folding, the first magmas that force their way from below into the prism of sediments are even more peculiar in character. These so-called "ultramafic" igneous rocks are extraordinarily low in silica and, in contrast to other ultra-basic rocks, contain several times as much magnesium as iron. They are rich in water, held chiefly in the mineral serpentine; analyses of the crystallized rock show between ten and 13 per cent of water. They rose into place and crystallized at low temperatures, so low, in fact, that they generally have produced little or no contact metamorphic changes in the sediments they invaded, though they appear in great volume. The outcrop of a single such body in the provinces of Camaguey and Oriente in Cuba covers about 5000 sq mi [HESS, 1938, p. 323]. On the southeast arm of the Island of Celebes, a similar nickel-bearing ultramafic intrusion covers an area of 8000 sq mi and is exposed to a depth of at least one kilometer [VAN BEMMELEN, 1949, p. 245]. Much discussion led to the concept of water-rich, high-magnesia ultramafics as the initial intrusive rock of young orogenic belts, called serpentines (or ophiolites, using an antiquated Greek name for the same rock). The chemical-mineralogic restriction of the concept furnished Hess the basis for a synthesis of existing areal information. Plotting impartially the distribution of these ultramafics as defined, he arrived at distribution maps which he presented before the 17th International Congress at Moscow [HESS, 1939]. Here is a second specific, powerful concept to add to that of geosynclinal troughs which later turn into orogenic mountain belts with deep roots.

Two conclusions of fundamental importance can be drawn from Hess' maps: (1) Ultramafic intrusions accompany the modern active orogenic belt from Halmahera through eastern Celebes, around the Banda Sea through Sumatra into Burma, where according to the important recent paper by EVANS and CROMPTON [1946] the great submarine belt of negative anomalies continues on land. (2) Comparable belts lie farther inland, corresponding to earlier geosynclinal zones. The most striking of these is that of the Urals.

Hess has shown that the relation between ultramafics and geosynclinal belts in their earliest stage of compression holds good in many parts of the Earth. It is a concept as trustworthy as many in the young science of geology.

If the generalizations with which we are operating mean what they seem to mean, then the formation and filling up of geosynclinal furrows and the early appearance of ultramafic intrusions and orogenic folding proceeded alike in young orogenic belts that run along the contact of continent and ocean, and in belts that lie between two continents. We shall see later that ultramafics are present also in an orogenic belt that rises straight from the floor of the ocean.

These are far-reaching conclusions that are stated with all due caution: the validity of these concepts remains yet to be proven by their ability to fit new facts as knowledge expands and becomes more precise.

Chemically, the ultramafics are most closely related to the peridotitic rock material that underlies the basalt in the lower part of the Earth's crust, but enriched in water and greatly enriched in magnesia. It looks as if this material gets started first and travels farthest, for in some cases, as for example, in Cuba, serpentinized peridotite rose high enough to come to rest in unconsolidated sediments. It looks, in fact, as if failure had started near the base and led to the downbending of the geosyncline; as if quantities of water and possibly hydrogen and other bases had been liberated in, or added from below to, the crust and through differential melting bodies of relatively mobile magma had formed; that driven by differential pressures in the failing crust and their own buoyancy, these bodies wormed upward through the crust, as salt and gypsum do in the piercement structures of folded sediments.

But the most telling fact about them is that they cease to rise when failure extends across the whole thickness of the crust and crumpling becomes general.

Metamorphism and granite as products of mature orogeny

This takes us to the third great concept to be discussed in this address: that of the metamorphic core of orogenic belts, which is riddled with granite bodies. This is something very different from the simple statement that "granite" forms the core of folded mountain ranges, which is one of the oldest of all generalizations in geology.

Metamorphic rocks and, of course, the plutonic rocks, are petrographically so utterly different from the sedimentary rocks on which the geological time sequence and all fundamental tectonic concepts are based, that the men who influenced the thinking on regional tectonics left their study to mineralogists and igneous petrographers. For a century they were simply thought of as "old," "Pre-Cambrian."

The discovery of fossils in true metamorphic rocks was one of the great events in the development of geological thought; their presence is one of the most crucial facts of geology; yet it is still so little known among geologists that one is greeted with amused smiles when one suggests to graduate students that they look for fossils in coarse mica-schists or sillimanite gneisses. The most authentic and authoritative modern book on metamorphic rocks does not mention fossils in the index. In the Alps, in the Scottish Highlands, in New England, to quote a few striking examples, fossils suddenly proved that great thicknesses of metamorphic sediments are of the same age as the formations in the unmetamorphosed marginal belt. The heart of orogenic deformation lies in the metamorphic core of the geosyncline.

The concept of a zone of metamorphism and plutonic activity as an integral, and actually the most crucial, part of a geosynclinal belt is still far from being formulated in such a manner as to permit impartial plotting of pertinent facts, because only long trial and error can tell what is pertinent. The reason lies, of course, in the devilish complexity with which metamorphosed sediments and plutonic rock bodies are enmeshed in this belt. Yet slowly a few tangible aspects of the metamorphic terranes are being formulated that can be represented on maps as positive knowledge, free from personal interpretation, and then progressively integrated to give fundamental insight.

The first is the recognition of zones of progressive regional metamorphism, characterized by specific minerals. The change from one into the other is rather abrupt so that their boundaries can be represented on maps by lines, the so-called isograds. The recognition of the existence of traceable belts in rocks of one orogenic unit seems to go back to a paper by G. Barrow on an area in the Southeastern Highland of Scotland, published in 1893. Barrow's method of plotting metamorphic zones was applied further to the highlands of Scotland by E. B. Bailey (who used different mineral indices), C. E. Tilley, W. Q. Kennedy, and other. In Scandinavia, it was systematically developed by V. M. Goldschmidt, T. Vogt, and their followers. T. F. W. Barth and Robert Balk have demonstrated the usefulness of this approach in a classical paper on the metamorphic rocks of Dutchess County, New York.

These and parallel investigations in other parts of the world are showing conclusively, I believe, that the pattern of belts of increasing metamorphism is largely independent of the structure pattern; they curve around areas of highest temperature. It is in these areas that the plutonic bodies lie, the great masses of granodiorite and granite. It is quite human to blame these granite bodies for the metamorphism. Paraphrasing the old fallacy, we say "juxta hoc, propter hoc."

Not everyone has thought thus, however. Over a century ago, the Norwegian geologist Keilhau concluded from his observations on metamorphic terranes that two stages of transformation can be recognized: (1) mere recrystallization due to increased temperature and pressure; and (2) chemical transformation by the removal of some substances and the introduction of others leading to granodiorite or granite as end product. He used the terms "metamorphism," "metasomatism," and "granitification." This view was adopted by the French school of petrographers, Durocher, Delesse. In 1861, Delesse declared that the plutonic rocks form at the expense of metamorphic rocks; they represent the extreme end of metamorphism; they are the effect and not the cause of metamorphism [WEGMANN, 1950].

That view was widely considered heretic then and it is still so, especially since the triumph of the Rosenbusch school of microscopic petrography. But the old heresy is rampant again. Like all such subversive movements, it confuses the fundamental issue by premature arguments about the nature of the processes that might be called upon to explain metasomatism and granitification, or, as we call it now, granitization.

Meanwhile, in the last three decades, careful studies in many regions have proven beyond reasonable doubt that granitization, the metasomatic transformation of sediments and enclosed lavas has taken place on a large scale. That is recognized by most petrographers. Whether granitization is possible is, therefore, not the main issue. Nor is there any question about sediments having been reduced locally to the molten condition in that process. J. J. Sederholm has described and documented all stages of that process and given them technical names.

The basic question is this: Is there in the Earth's crust a primary shell of granite, that lies above the basalt as an inheritance from the primeval days of the crust, which holds in itself the potentialities of all the thermal and chemical effects we observe in metamorphic belts that need to be only released? Or is granite a by-product of orogenic activity, that is, was there no granite when the crust came into being, forming a world-wide carapace that surrounded the Earth? Or, to put it succinctly, is the granite of orogenic belts "made" or "remade"? I need not underscore the importance which this question has for geophysicists as well as geologists.

It is futile to look to the microscope and the experimental laboratory for an answer to this question. If an answer is possible at all, it must come from the evaluation of all available field evidence. It is significant, that the heresy is practically limited to men whose life's work lies in the field.

Here, careful attention to detail has brought about a change in map representation, a gain in precision of symbolic recording. This is illustrated by two maps taken from a paper by SAKSALA [1937, Fig. 3, p. 279; Tafel 1, opp. p. 292] in the fifieth anniversary volume of the Geological Commission of Finland. It shows the same region in the Pohjanma district of Finland in an earlier and a recent map representation. The earlier map shows in solid color the areas where granite predominates; the later map represents diagrammatically what is seen. When each of the solid areas was searched carefully, bands of schist and associated rocks were found to extend into, to lie within, and to pass through these areas without much derangement. The other map shows the same area on a smaller scale as part of the larger regional picture (in the northeast). This relation between intrusive and intruded sediments is seen to be typical wherever deeply eroded metamorphic zones are studied.

The relation can be neatly illustrated by a specimen in my collection [BUCHER, 1948, pp. 125-126, and pl. 1 opp. p. 126]. This block of rock, about a foot square, consists of very different material. In the space of inches, it shows layers of dark shale that have been invaded by a lighter gray, crystalline material in the manner which is characteristic of granite on the scale of miles. The "intrusive" sends off numerous shoots that look as if they had wedged between and split apart the layers. Others cut across the shale as dikes in different directions. One body of shale is completely surrounded by the intrusive, yet maintains its orientation with reference to the structure of the rest. That is the most striking feature of innumerable belts of metasediments that occur in granite intrusives of this kind.

In the scale model of the hand specimen as in its counterparts in nature, this behavior is difficult to understand if it is assumed that the intrusive has been forcibly introduced from the outside.

In my specimen, nothing of the sort has happened. The apparent intrusive is merely the substance of the shale recrystallized as hornfels consisting of diopside and garnet. The specimen was collected from within a few feet of the contact with a basaltic sill that baked the shale for some distance. Slight differences in chemical composition and porosity and the presence of mud cracks in the shale account for the differential recrystallization.

If the similarity of this cubic foot of rock to the cubic miles of granite in metasediments is more than an analogy, then it can give a satisfactory explanation for the first of several field facts:

Granites always appear where sediments disappear. The manner in which they appear in this type of granite is wholly unlike what one might expect from a foreign mass that made room for itself by breaking away the opposing wall piece by piece like an army of miners stoping its way into an ore body.

But this is only one type of granitoid intrusion. It is called "syntectonic," because the rocks of this type show the effects of intense deformation in their larger relations as well as in the coarse cataclastic and foliated texture of a hand specimen, indicating that they reached their position and character while the directed regional orogenic pressure was active. All that was said of wide belts of metamorphism culminating in a belt of intimately intermixed granitoid and metasedimentary materials surrounding the granite bodies themselves, applies to this type.

The other type presents the opposite picture. It cuts across all earlier rocks in a way that makes the map picture look as if someone had put a hot iron against a delicate wax fabric, leaving an irregular, featureless splotch. They produce only local contact metamorphic aureoles and no migmatite belt. They tend to show a broad porphyritic border zone and miarolitic texture. They contain within them and form near them the so-called "pneumatolytic concentrates" of pegmatites and ore veins. They must have reached their place after the directed tectonic pressure had ceased; hence they are called "post-tectonic."

Here is a second concept that stands for tangible field experience that can be expressed on maps. In a general way, this distinction is reflected, for example, in HOLMQUIST'S [1936, pl. 4, opp. p. 326] map of the Swedish Archean, where the late granite bodies, shown in black, contrast with the earlier ones, especially the Early Archean granites shown by a pattern of crosses.

A third significant field fact can be formulated when the third dimension is taken in consideration. Wherever regional mapping and analysis have gone far enough, it is found that the post-tectonic granites dominate at the highest levels reached by intrusives and there is less of them as one goes down to deeper regions.

There is still another field fact of great importance. As one goes from the earlier to the later syntectonic plutonic bodies, one goes to materials that are progressively richer in silica and potash, that is, from diorites and granodiorites to true granites.

The last of the general field facts is the most important: the streaming structure pattern of the young orogenic mountains is identical in principle with that of the metamorphics of the Pre-Cambrian areas. Here we can point once more to the growing insight that comes from synthesis of mappable knowledge. The first of two slides shows the tectonic pattern of a part of the Sierra Nevada, as compiled by LOCKE, BILLINGSLEY and MAYO [1940, pl. 1, opp. p. 513]. Compare this with WILSON'S [1950, Fig. 1, p. 103] map of the western part of the Grenville province.

The general streaming plan is the same and bears out the conclusion that the Pre-Cambrian basement, as shown in the great shields of the continents, is but the deeply eroded aggregate of earlier geosynclines transformed into orogenic belts.

To get our bearings, let us return to what was said concerning the ultramafic intrusives. They appear at relatively low temperatures, rich in water, at the very beginning of orogenic compression. It looks, as previously noted, as if failure had started near the base of a rather strong crust and had led to differential melting and squeezing up of the mobile melt, which rises even into unconsolidated sediments, and arrives with remarkably low temperatures.

As failure becomes general, more silicic intrusives appear, such as diorites and granodiorites. Progressing with time and upward through the crust, the advancing intrusive material carries more and more silica. Silica, soda, and to some extent potassium and other elements impregnate the sediments for miles, even tens of miles beyond the outlines of the intrusives themselves. This suggests a transfer of water and diverse elements in the gaseous state in quantities apparently out of proportion to the volume of the intrusives themselves. When the orogenic pressure dies down, the products richest in silica, potash, and volatiles keep on gathering and rise once more as high as the ultramafics did at first.

I believe that this sequence fits best into the following speculative picture. Let us assume that (a) olivine basalt of the central Pacific type is the original material of the crust; and (b) that the crust fails after the fashion of a cylinder of rock in David Grigg's experiments under high confining pressure. He found that rock flowage and the formation of fractures go hand in hand at all the pressures reached by him. If this were true across the whole thickness of the crust, the proces of "crush-flowing" would do two things: (1) it would open avenues of escape to water, hydrogen, and other elements in the gaseous state to be distilled off from the materials of the subcrust and the lowest part of the crust to enter the higher parts, carrying with them great excess of heat; and (2) it would produce heat through friction, and thus provide at least some of the energy required to liberate the materials and boost them in their upward progress [COOKE, 1927; 1948, especially comments on shearing, p. 35].

The elements that enter into metasomatic processes that are involved in the metamorphism and granitization of sediments and possibly in the formation of granite itself, are primarily sodium, potassium, alumina, and silica. If moderate amounts of these elements were distilled off (if I may use this expression) from the material of the lower part of the Earth's crust, that is, olivine basalt, the latter would assume the character of periodotite, that is, the rock material that seems to form the bulk of the subcrustal silicate mantle of the Earth and is chemically like the material of the average stony meteorite (see Table 1). In other words, a differentiation would be produced dynamically in the lower part of the crust that would provide much of the material of the so-called "emanations" needed for regional metamorphism and the rest magma that appears in the post-tectonic intrusives.

Table 1--Comparison of analyses of the silicate phase of stony meteorites, Hawaiian olivine basalt and augite andesite of western Pacific Island arcs

Constituent	Silicate phase of stony meteorites (107 analyses)[a]	Olivine basalt (Hawaiian Archipelago) (53 analyses)[b]	Augite andesite (Marianas, Philippines and Halmahera) (17 analyses)[c]
	pct	pct	pct
SiO_2	46.26	48.35	55.87
Al_2O_3	3.45	13.18	18.13
$FeO + Fe_2O_3$	14.95	11.43	7.69
MgO	27.56	9.72	3.60
CaO	2.90	10.34	8.02
Na_2O	1.10	2.42	3.37
K_2O	0.25	0.58	1.92
H_2O	0.58	Not given	1.15
TiO_2	0.12	2.77	0.35[d]
P_2O_5	2.31	Not given	0.13[d]

[a] BROWN, HARRISON and PATTERSON, CLAIR, The composition of the silicate phase of stony meteorites, J. Geol., v. 55, pp. 405-411, 1947.
[b] MacDONALD, G. A., Hawaiian petrographic province, Geol. Soc. Amer., Bull., v. 60, pp. 1541-1596, 1949.
[c] VON WOLFF, F., Der Vulkanismus, v. 2, pt. 1, pp. 162-178, Stuttgart, 1929.
[d] Based on seven analyses only.

That chemical alteration under shear may play a very important part in the Earth's crust was pointed out, for instance, by BRIDGMAN [1936, pp. 668-669]. VERHOOGEN [1949] shows how complicated and far from obvious the relations between the gaseous and other phases are under high temperatures and pressures when viewed comprehensively in the light of modern thermodynamics, especially in an open system such as this hypothesis implies.

In a recent paper on granite and metamorphism, DALY [1949, p. 768] points out, as he puts it, that "the metasomatic hypothesis (of granite origin), expressed in any of its chameleon forms, recognizes no need for the emanations to choose some routes upward rather than others."

Therefore, it is "quite unable to account for the utter lack of granite or rhyolite within the sial-free Earth-sector, capped by the deep Pacific Ocean, which covers nearly one-quarter of the globe" or for the absence of any signs of granitization since the beginning of Cambrian time over the three million square kilometers or more of the Canadian Archean shield.

Instead of a crushing argument against the formation of granite as an immediate consequence of orogeny, this statement is the most potent argument for it. The question is no longer: Why is granite absent from the ocean floors, but why is granite present in the continents? It looks very much as if the answer were: Because granite comes into being in orogenic belts.

But that runs counter to much that is deeply ingrained in our geological thinking. Let us view two such ideas. We have been taught that: (1) Granite forms a thin outer shell (some ten kilometers thick) above the basalt in all oceans except the central Pacific. Maurice Ewing and Frank Press have shown recently that the dispersion of surface waves can be accounted for as the result of the boundary effect of water resting on olivine basalt and that the corresponding effect due to the presence of an interface between a granitic and a basaltic shell would give results much at variance with the actual observations. In the northern Atlantic Ocean, Ewing has actually determined the velocity of seismic waves at the base of recent sediments as that of olivine basalt. There is no granite there, and by extrapolation, perhaps not elsewhere on the main parts of the ocean floors. (2) But do not all sediments ultimately rest on a granite foundation? No one has ever seen primary granite and its effusive derivatives as a crustal unit by itself, brought up by faulting or updoming, in the way basalt appears wherever it is brought to light in the oceanic areas. Wherever it is seen, granite is always "intrusive."

But how about the earliest sediments of the oldest Archean? In looking through the literature, I was deeply impressed with one peculiarity of the earliest sediments in all parts of the world: They abound in sedimentary iron ore deposits, in basic pillow lavas, and in fine-grained siliceous sediments, recognized as cherts in some regions, called fine-grained, bedded, unique siliceous volcanic rocks in others. Independently, in many regions, the suspicion has been voiced that the coarser "quartzites" associated with such deposits are recrystallized cherts. Backlund has recently given a graphic picture of the texture and structure of fine-grained siliceous rocks of Sweden, the "leptites and hälleflintas," that makes it appear probable that they are slightly altered cherts.

Iron oxide and colloidal silica are typical products of the weathering of basalt. If the original crust had consisted of granite, then coarse-grained quartz sandstones and feldspar-rich sandstones should be the most common sediments in the earliest part of the Earth's recorded history.

On the face of the available evidence, it is not unreasonable, therefore, to assume that the original crust of the Earth consisted of alkaline basalt and that the light sialic continents are the products of orogeny. If the orogenic belts came into being in the manner suggested by Quirke's and my experiments, by local failure of the crust, the clustering of orogeny around south-central Asia would be explained. The growth of continents by addition at the margins would follow. Of the outermost of these, the ones that stand far out on the ocean floor would hold the final clue to our understanding.

The great submarine range that extends north from Guam through the Mariana and Bonin Islands in the western Pacific is the greatest example. Thanks to HESS' [1948] beautiful map of the western Pacific we know what this great submarine welt would look like if there were no water and we could view it from the vast lowland to the east which is the ocean floor. We would see a great double mountain range, comparable with the Himalayas in length and width and to some extent even in ground plan. West of the center of this mighty mountain welt another great lowland extends for over a thousand miles, with depths as great and with positive gravity anomalies apparenty as high as those in the Central Pacific. Surely this mighty mountain range stands on the floor of the Pacific as the Himalayas stand on the continental platform. It is high because it was raised and not because the lowland to the east and west was lowered.

But if we were to cross the thousand miles west of this submarine welt, we would see the great ranges of the Philippines and Ryukyus rise in precisely the same way. Standing there on the ocean floor, we would realize that the fundamental question of crustal deformation is not, "Why are the ocean floors low?" but, "Why are the continents high?" This is the counterpart of the question we asked before: "Why is granite absent from the ocean?" The answer may be the same: "Because their changed chemical composition and their elevations are products of orogenic deformation." Perhaps this is not so silly as it sounds at first.

Why should anyone suggest that the Mariana-Bonin welt, which bears all the earmarks of having been raised above the deep-sea level, marks a major discontinuity between the Pacific Basin type of crust and the surrounding continental area, the so-called "andesite line"? The writer knows of two reasons: one is petrographic, the other geophysical. The petrographic argument is expressed in the name "andesite line." Inside this line within the Central Pacific, olivine-rich alkali basalt reigns supreme among the island lavas; outside, pyroxene-andesite. Why?

Pyroxene-andesite is the most common extrusive rock next to basalt. Since it consists of a mixture of pyroxene and plagioclase crystals, it cannot well have been derived from a basaltic parent magma by gravitational differentiation or by the expulsion of residual fluid from a partly crystallized basalt. This has led to the conclusion, in the words of SHAND [1947, p. 279], that "the production of pyroxene-andesite depends upon the acidification of basaltic magma by reaction with the siliceous rocks of the continental regions." This is where the "continental regions" come in. The word andesite is thought to call for "siliceous rocks of the continental regions." But does it? The essential part of the sentence just quoted relates the production of pyroxene-andesite to "the acidification of basaltic magma." The advocates of granitization present evidence that suggests that silica, alumina and alkalis enter the upper part of the crust from farther down, and that they do so only in the active orogenic belts. Table 1 shows that it is the enrichment in these materials that distinguishes an augite andesite from a Hawaiian olivine basalt, just as impoverishment of the same materials changes a basalt into a peridotite. If it is true that these changes are induced by orogenic deformation, then the andesite line may mark merely the borderline between passive oceanic basalt areas and active orogenic belts. In that case, there should be an andesite line in the Atlantic at the edges of the two active orogenic belts that lie in it. There is. Andesite abounds among the lavas of the Antillean and South Antillean arcs, each of which exhibits the unique combination of properties which is so characteristic of the active orogenic belts of the Pacific: a deep-sea trench, an asymmetrical welt with lines of volcanoes on its back, the characteristic double curve of gravity anomalies, and its quota of earthquakes, including some from considerable depths.

When followed through, this line of reasoning leads one to the same doubt concerning the venerable dogma of the Earth's primordial sial shell we met with before. If basalt can be changed to andesitic magma by the introduction of silica, alumina, and alkalis into the upper part of the crust along active orogenic belts, then after erosion and sedimentation, introduction of the same materials into geosynclinal sediments (which are relatively richer in silica) may lead indeed eventually to granitization.

The geophysical argument for the traditional view concerning the nature of the andesite line arises from GUTENBERG'S [G. S. A. symposium, 1949] important observation that surface waves passing for longer distances along the boundary of the Pacific basin as it is determined by the andesite line "show a noticeable loss of energy occasionally as high as 90 per cent." From this Gutenberg drew the conclusion that in this way it is possible to locate approximately the boundary of the Pacific structure. Is it not possible that the physical condition of the Earth's crust within an active orogenic belt is responsible for this effect? If the deep sea floor west of the Mariana-Bonin arc really consists of continental material, fundamentally different from the ocean floor east of it, then seismic refraction studies will give decisive proof. They should be made soon.

With this outlook, the geologist meets the geophysicist. Our problems are the same when we think about the Earth's crust. The facts concerning the three-fourths of the surface that are buried beneath ice and water, and all that lies below the first few miles of rock can be investigated only by geophysical means.

Through the indefatigable efforts of such men as George Woollard, the rapidly accumulating gravity and magnetic data concerning the continents are being collected and integrated on maps. Maurice Ewing leads a band of enthusiastic workers who gather facts concerning the nature of the ocean floors, and assemble them in graphic form on maps.

The students of terrestrial magnetism are focusing their attention on the measuring of local anomalies and their changes. The seismologists are probing the structure of the Earth, suspecting that the concentric layer structure they have assumed for the crust is too simple. Oceanographers are getting deeper and deeper into the mantle of modern sediments and the record they hold. In this picture of detailed investigation, geology holds a modest, but important field. Before the theoretical geophysicist can set up equations, he must know in what manner the crust fails, in what pattern, with what internal consequences, and at what rate. For many decades, the prime task of the geologist will be to continue the detailed study of the visible realities of the Earth's crust. The individual workers do so generally for purely local reasons, mostly economic. They must be taught to think of the larger purpose to which their findings will ultimately be put. They must know what to represent on maps and in the text and how to do it most effectively; and their results must be painstakingly analyzed from the larger points of view, rechecked, tabulated, and graphically drawn on maps. There is no other way to find out what is real and what is imaginary in the things that ultimately will go into the great final equations of which geologists and geophysicists dream.

It is the prime function of the International Union of Geodesy and Geophysics to further all efforts of integration and of cooperation. Our National Committee, the AGU, has a proud record of promoting such cooperation and the resulting synthesis. Let us continue the good work.

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ON A METHOD OF SEMI-RIGOROUS ADJUSTMENT OF TRIANGULATION NETWORKS

B. Goussinsky

Abstract--This paper describes a method of adjustment suitable for rapid and relatively extensive triangulation surveys, which require economy in adjustment combined with solution of all condition equations as simultaneously as possible. Without being absolutely rigorous from the point of view of least squares, the method is, however, only a simplification of the latter, and is, to the author's opinion, most suitable for adjustment of triangulation networks carried out for geophysical or topographical surveys.

The idea upon which the method is based, is, in principle, the same as the one described [GOUSSINSKY, 1948] with regard to traverse adjustment. All condition equations are arranged, as usual, in two classes: (1) angular condition equations, and (2) side condition equations. The principle of the method consists in carrying out the adjustment in two steps. First, all the angular conditions are adjusted; in the second stage only the side equations are dealt with, but in such a way that no special condition equations need be required in order to leave undisturbed the angular conditions already satisfied in the first stage of the adjustment.

The first stage need not be described in detail; all independent conditions for angular closure are collected and adjusted in one set. There is then a single set of bearings for all sides of the network which has become, so to speak, rigid in orientation. This means that if we start from a side having a certain bearing, every other side will have only one value for its bearing, regardless of the route by which it has been obtained from the base.

The network is, however, not yet completely adjusted, since the lengths of the sides are not definitely fixed. If we start from a base having a fixed length, each of the other sides will have more than one value, this being dependent upon the route from the base through which its length is computed. This discrepancy is eliminated in the second part of the adjustment in order to render the network absolutely rigid. With angles adjusted for angular closures the side equations are taken up.

In the second stage of the adjustment, it is assumed that the corrections to be applied to the two directions at the extremity of a side are equal in sign and magnitude. This is the only deviation from the rigorous procedure which was allowed in the adjustment, and due to which it has been possible to split the work into two parts. Geometrically this can be visualized as follows. In order to satisfy the sine formula each side is turned, as a rigid bar, by a certain angle, and its two reciprocal bearings receive thus the same correction in sign and magnitude. In the process of adjustment this is obtained by denoting by the same symbol (letter or figure) the corrections applied to the two directions of a line.

It is easy to see that after the second adjustment, the angular conditions, already satisfied in the first adjustment, remain undisturbed. In a triangle or any other figure, the sum of corrections to its angles, resulting from corrections to the respective directions, is equal to 0, and the angular closure is, therefore, not affected.

As one can see, the method of adjustment just described is applicable only when observations are carried out by directions, not by angles, and when there are no enforcing conditions.

The example given below shows the comparative results of two adjustments of the network shown in Figure 1, (1) by the method given above, and (2) by a rigorous simultaneous adjustment for all condition equations. Figure 1 is a quadrilateral with two diagonals and an overlapping central polygon. There are 11 sides, all observed both ways, and seven points. The adjustment comprises six angle and two side equations. The corrections for the angular adjustment are denoted by numerals from one to 22. For the side adjustment, the corrections are denoted by letters, from a to ℓ. Their number is that of the lines in the figure, that is, 11, and the two directions at the extremities of a side are denoted by the same letter.

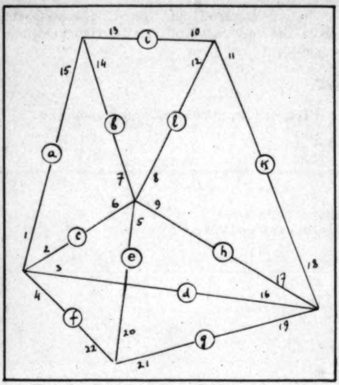

Fig. 1--Simple triangulation network

In Table 1 below, Col. 1 and 2 show the corrections obtained in the two successive stages of adjustment. Col. 3 gives the sum of these corrections, that is, the total correction for each direction, whereas in Col. 4 the corrections resulting from the rigorous adjustment are shown. Finally, in Col. 5 are the differences between the values in Col. 3 and 4. These, as one can see, are very small when considered in the light of individual corrections (about 0".1 on the average), and so is also the difference between the sums of the squares of corrections in the two adjustments. These are 12".305 and 11".930, the second being the minimum resulting from the rigorous adjustment.

It might be of interest to note that the corrections obtained from the angular adjustment show a symmetry similar to that resulting from the side adjustment; namely, the corrections at the ends of each line here are also equal in magnitude, though of opposite signs. This is easily explained seeing that all observations have been assumed to be of equal weights.

Table 1--Angular adjustment of the network

Angle	1	Line	2	3	4	5
	"		"	"	"	"
1	+ 0.493	a	+ 0.676	+ 1.169	+ 1.003	+ 0.163
2	+ 0.569	c	+ 0.012	+ 0.581	+ 0.435	+ 0.146
3	- 0.813	d	- 0.626	- 1.439	- 1.503	+ 0.064
4	- 0.249	f	+ 0.290	+ 0.041	+ 0.058	- 0.017
5	- 0.316	e	- 0.362	- 0.678	- 0.445	- 0.233
6	- 0.569	c	+ 0.012	- 0.557	- 0.342	- 0.215
7	+ 0.427	b	- 0.603	- 0.176	+ 0.088	- 0.264
8	+ 0.346	l	- 0.150	+ 0.196	+ 0.356	- 0.160
9	+ 0.112	h	+ 0.048	+ 0.160	+ 0.343	- 0.183
10	- 0.920	i	+ 0.087	- 0.833	- 0.837	+ 0.004
11	+ 1.266	k	+ 0.203	+ 1.469	+ 1.433	+ 0.036
12	- 0.346	l	- 0.150	- 0.496	- 0.595	+ 0.099
13	+ 0.920	i	+ 0.087	+ 1.007	+ 0.975	+ 0.032
14	- 0.427	b	- 0.603	- 1.030	- 1.049	+ 0.019
15	- 0.493	a	+ 0.676	+ 0.183	+ 0.074	+ 0.109
16	+ 0.813	d	- 0.626	+ 0.187	+ 0.217	- 0.030
17	- 0.112	h	+ 0.048	- 0.064	- 0.153	+ 0.089
18	- 1.266	k	+ 0.203	- 1.063	- 1.105	+ 0.042
19	+ 0.565	g	+ 0.421	+ 0.986	+ 1.045	- 0.059
20	+ 0.316	e	- 0.362	- 0.046	- 0.238	+ 0.192
21	- 0.565	g	+ 0.421	- 0.144	- 0.250	+ 0.106
22	+ 0.249	f	+ 0.290	+ 0.539	+ 0.489	+ 0.050

The advantage of the method consists in its being much easier to carry out than the rigorous adjustment. In the first stage of the adjustment the coefficients of all the unknowns in the angle condition equations are invariably equal to unity. The formation of the normal equations and their solution is, therefore, relatively easy. The second part of the adjustment, where the coefficients of the side equations are odd numbers, is made simple due to two factors. (1) the number of unknowns is reduced by half; and (2) the number of condition equations is small.

When comparing this method with the commonly used figure-by-figure adjustment, one might say that, whereas the latter consists of parts that are all situated in the same horizontal plane, the former consists of two layers of adjustment superimposed vertically. The superiority of the demonstrated method of adjustment is obvious; there being no enforcing conditions, which accumulate as one proceeds with the work, the gain is in both accuracy and speed. The method may justly be called semi-rigorous. It would seem to be the best substitute to the rigorous procedure, whenever the latter is not absolutely indispensable.

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PUNCH CARD METHOD OF CONVERTING GEOGRAPHIC COORDINATES TO UNIVERSAL TRANSVERSE MERCATOR GRID COORDINATES

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Abstract--The purpose of this paper is to show how to compute Universal Transverse Mercator Grid Coordinates from geographic coordinates by use of the punch card and Universal Transverse Mercator Tables. The general mathematical equations used to obtain Universal Transverse Mercator Coordinates are reduced to combinations of the basic arithmetic operations on the punch card followed by an actual problem of conversion from geographic coordinates into Universal Transverse Mercator Grid Coordinates. The results of this paper lead to the conclusion that the punch card method of computation can readily be adapted to numerous geodetic, engineering, mathematical, and scientific problems.

Introduction--During 1948, one of the activities of the Geodetic Division of the Army Map Service has been to convert geographic positions within the United States into Universal Transverse Mercator Grid Coordinates by means of the punched card.

Although the techniques of the punched-card method have been generally known to the statistical and business world for half a century, the application of the punched card for the solution of mathematical, scientific, and geodetic problems has not been extensively utilized by the mathematician, scientist, and engineer until the last decade.

It is believed that the punched card method in making mass engineering computations was first undertaken in 1938 by the Mathematical Tables Project with the cooperation of the Military Grid Project which were part of the Works Projects Administration in New York. These projects were sponsored by the War Department in cooperation with the Bureau of Standards and the United States Coast and Geodetic Survey. After the termination of the Works Projects Administration in July 1942, the Military Grid Project was reorganized and placed in operation in New York under the Chief of Engineers and in due time was incorporated into the Army Map Service. During the period 1941-1944, computations of Military Grid Coordinates and construction of geodetic tables, both Lambert and Transverse Mercator, had been performed by means of punched cards.

Since this paper limits itself to the computation of the Universal Transverse Mercator Grid Coordinates from geographic coordinates by use of the punched card and Universal Transverse Mercator Tables, the problem will be indicated on the punched card in general terms and then a solution of a specific problem in conversion will be computed on the punched card. The geographic position for which the latitude is 41° 31′ 51.″718 and longitude is 96° 08′ 09.″858 is used in the example on the punched card to indicate the conversion to Universal Transverse Mercator Grid Coordinates. The following equations are used.

$$N = (I) + (II)p^2 + (III)p^4 + A_6$$

$$E = FE \pm [(IV)p + (V)p^3 + (VI)p^5]$$

in which $p = 0.0001\Delta\lambda''$ and $FE = 500{,}000$ meters.

The notations and definitions of these formulas are explained in AMS Technical Manual 19. Tables used for this computation are taken from AMS Technical Manual 6.

Discussion--To utilize the punch-card method in mathematical computation, it is necessary to break down the basic equations into their component parts. After each formula is broken down, a card layout is drawn, indicating the fields or columns needed for the required steps in the computation. The card may be considered as a computing form with the equation arranged to show the successive steps required for the mathematical treatment. Although card forms were printed for Cards 1 and 5 (see Fig. 1 and 2) for the Universal Transverse Mercator Grid computations, this is not absolutely necessary since any available cards can be used in conjunction with a sample card layout form. Cards 4A, 2, 4B, and 3 (see Fig. 3, 4, 5, and 6) were not printed for this computation. In

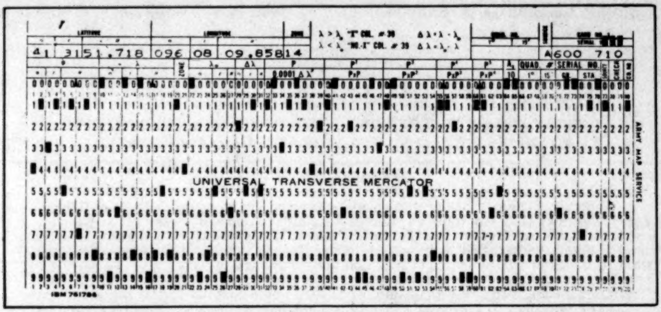

41 31 51.718 096 08 09.858 14
A 600 710
UNIVERSAL TRANSVERSE MERCATOR
ARMY MAP SERVICE

Fig. 1--Card 1 showing manually punched latitude, longitude, serial number, source, card number, and the computation of P and the powers of P

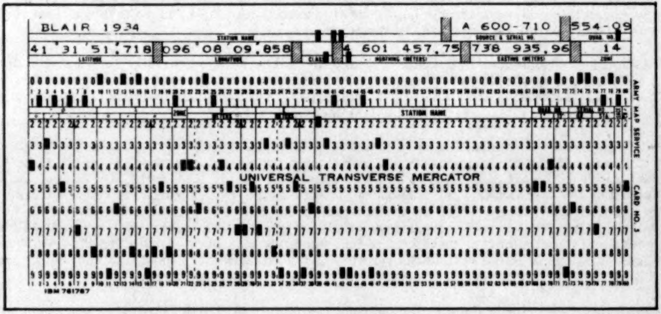

BLAIR 1934
A 600-710
554-09
41 31 51.718
096 08 09.858
4 601 457.75
738 935.96
14
UNIVERSAL TRANSVERSE MERCATOR
ARMY MAP SERVICE
CARD NO. 5

Fig. 2--Card 5; final card contains manually punched latitude, longitude, station name, serial number, and source; computed Universal Transverse Mercator coordinates; and indicated zone and quadrangle number

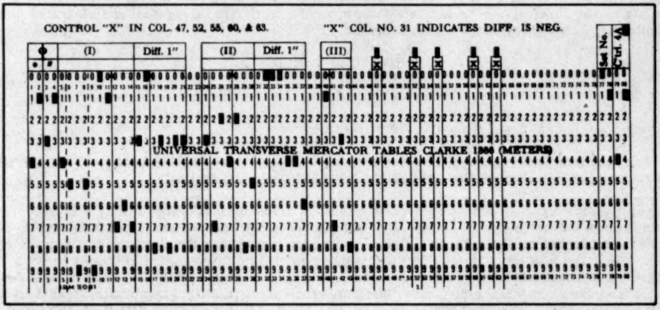

CONTROL "X" IN COL. 47, 52, 55, 60, & 63.
"X" COL. NO. 31 INDICATES DIFF. IS NEG.
(I)
Diff. 1"
(II)
Diff. 1"
(III)
Set No.
UNIVERSAL TRANSVERSE MERCATOR TABLES CLARKE 1866 (METERS)

Fig. 3--Card 4A; U. T. M. Tables punched into card used to interpolate factors required for northing computation

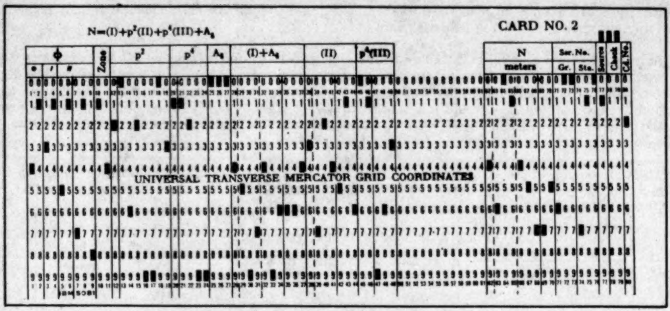

N=(I)+p²(II)+p⁴(III)+A₆
CARD NO. 2
φ
° ′ ″
Zone
p²
p⁴
A₆
(I)+A₆
(II)
p⁴(III)
N
meters
Ser. No.
Gr.
Sta.
Source
Check
Cd. No.
UNIVERSAL TRANSVERSE MERCATOR GRID COORDINATES
IBM 5081

Fig. 4--Card 2; ϕ, zone, p^2, p^4, A_6, serial number, and source transferred from card 1; interpolated factors and computed northing

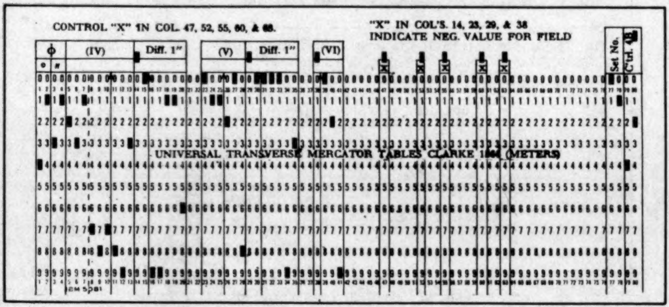

CONTROL "X" IN COL. 47, 52, 55, 60, & 63.
"X" IN COL'S. 14, 23, 29, & 38
INDICATE NEG. VALUE FOR FIELD
φ
° ″
(IV)
Diff. 1″
(V)
Diff. 1″
(VI)
X
X
X
X
X
Set No.
C'trl. 4B
UNIVERSAL TRANSVERSE MERCATOR TABLES CLARKE 1866 (METERS)
IBM 5081

Fig. 5--Card 4B; U. T. M. Tables punched into card used to interpolate factor required to compute easting

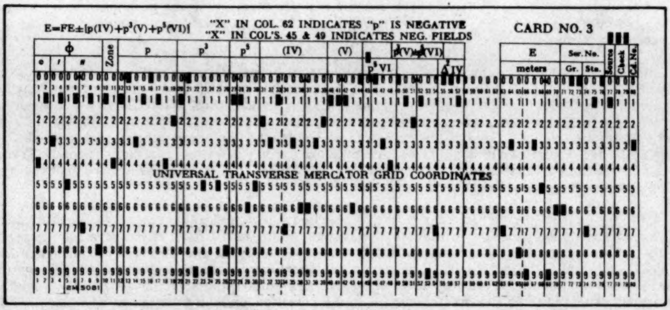
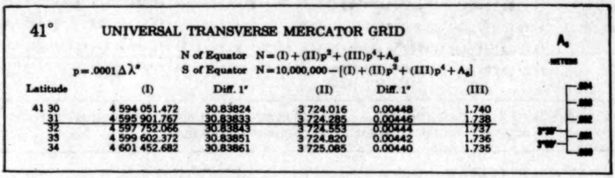

E=FE±[p(IV)+p³(V)+p⁵(VI)]
"X" IN COL. 62 INDICATES "p" IS NEGATIVE
"X" IN COL'S. 45 & 49 INDICATES NEG. FIELDS
CARD NO. 3
φ
° ′ ″
Zone
p
p³
p⁵
(IV)
(V)
p³(V)+p⁵(VI)
p⁵ VI
p³ IV
E
meters
Ser. No.
Gr.
Sta.
Source
Check
Cd. No.
UNIVERSAL TRANSVERSE MERCATOR GRID COORDINATES
IBM 5081

Fig. 6--Card 3; ϕ, zone, p, p^3, p^5, serial number, and source transferred from Card 1; interpolated factors and computed easting general the card layout is designed to serve as a guide to indicate the sequence required for mathematical treatment and wiring control panels.

In order to keep control of data and to identify individual geographic positions, serial numbers are assigned to each geographic position on source documents. The serial numbers are six digit figures. The first three digits identify the group and the last three digits identify the station in the group; for example, 600 710 would indicate station 710 in group 600. These serial numbers serve a dual purpose, to avoid confusion in cases of duplicate stations and to readily locate a station when necessary.

The conversion of the geographic positions into Universal Transverse Mercator Grid Coordinates consists of interpolation from AMS Technical Manual 6 Tables (see Fig. 7 and 8). These are punched into cards 4A and 4B (Fig. 3 and 5). Tables punched into the card are checked by second differencing (see Fig. 9). This check quickly identifies errors in the punched cards, missing cards, and cards out of order. It will be noticed that the cards are arranged according to an ascending sequence of the latitude.



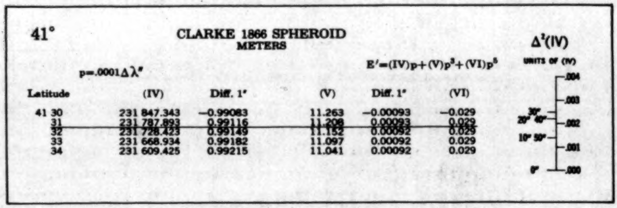
41° UNIVERSAL TRANSVERSE MERCATOR GRID

N of Equator $N = (I) + (II)p^2 + (III)p^4 + A_s$

$p = .0001\Delta\lambda''$ S of Equator $N = 10{,}000{,}000 - [(I) + (II)p^2 + (III)p^4 + A_s]$

Latitude	(I)	Diff. 1"	(II)	Diff. 1"	(III)
41 30	4 594 051.472	30.83824	3 724.016	0.00448	1.740
31	4 595 901.767	30.83833	3 724.285	0.00446	1.738
32	4 597 752.066	30.83843	3 724.553	0.00444	1.737
33	4 599 602.372	30.83851	3 724.820	0.00442	1.736
34	4 601 452.682	30.83861	3 725.085	0.00440	1.735

Fig. 7--Sample Universal Transverse Mercator Tables used to compute northing



41° CLARKE 1866 SPHEROID
METERS

$E' = (IV)p + (V)p^3 + (VI)p^5$

$p = .0001\Delta\lambda''$

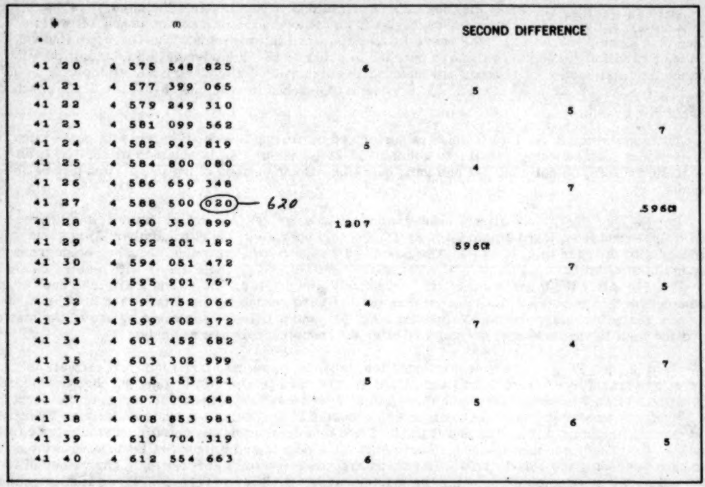
Latitude	(IV)	Diff. 1"	(V)	Diff. 1"	(VI)
41 30	231 847.343	−0.99083	11.263	−0.00093	−0.029
31	231 787.893	0.99116	11.208	0.00093	0.029
32	231 728.423	0.99149	11.152	0.00092	0.029
33	231 668.934	0.99182	11.097	0.00092	0.029
34	231 609.425	0.99215	11.041	0.00092	0.029

$\Delta^2(IV)$
UNITS OF (IV)
.004
.003
30"
20" 40"
.002
10" 50"
.001
0"
.000

Fig. 8--Sample Universal Transverse Mercator Tables used to compute easting

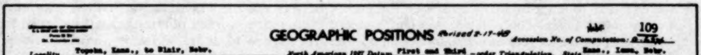
In order to prepare the punch cards for mechanical processing, the initial information, geographic coordinates, code number, source, and card number are punched manually from source documents (see Fig. 10) in designated columns of Cards 1 and 5 (Fig. 1 and 2) by two different operators. In addition to the preceding information, the station name is punched into Card 5. This punched information, exclusive of the station name, is checked by comparing Card 1 against Card 5 in the collator. Error cards, duplicates, and missing cards are isolated in this process. Error cards are checked against source documents and corrected. The station names are checked by listing and proofing against source documents.

After the manually punched information is checked, Card 1 (Fig. 1) is sorted with master cards on degrees and tenths position of minutes of longitude (columns 10-13). This operation separates groups of cards into belts according to the master card set-up for the Universal Transverse Mercator Grid System. The master card precedes the detail cards. (In this operation 20,000 or more cards are sorted in one operation.) After the cards are sorted, the zone and longitude of origin (λ_o) are transferred from the master card onto the detail cards by a gang punch operation. This information is checked by comparing results punched into the detail cards in the comparing unit of the reproducer. Cards falling within the overlap areas are isolated by sorting on a code number and a duplicate set of cards is made for overlaps. These extend 25 mi beyond the standard 6° zones.



ϕ	m	SECOND DIFFERENCE			
41 20	4 575 548 825	6			
41 21	4 577 399 065		5		
41 22	4 579 249 310			5	
41 23	4 581 099 562				7
41 24	4 582 949 819	5			
41 25	4 584 800 080		4		
41 26	4 586 650 348			7	
41 27	4 588 500 020 — 620				596CR
41 28	4 590 350 899	1207			
41 29	4 592 201 182		596CR		
41 30	4 594 051 472			7	
41 31	4 595 901 767				5
41 32	4 597 752 066	4			
41 33	4 599 602 372		7		
41 34	4 601 452 682			4	
41 35	4 603 302 999				7
41 36	4 605 153 321	5			
41 37	4 607 003 648		5		
41 38	4 608 853 981			6	
41 39	4 610 704 319				5
41 40	4 612 554 663	6			

Fig. 9--Second difference check; automatic second differencing of tables from punched cards; function for argument of 41° 27′ was punched incorrectly in card



GEOGRAPHIC POSITIONS Revised 2-17-48 109

Locality Topeka, Kans., to Blair, Nebr. North American 1927 Datum. First and Third -order Triangulation. State Kans., Iowa, Nebr.

STATION | LATITUDE AND LONGITUDE | AZIMUTH | BACK AZIMUTH | TO STATION | DISTANCE

Hale (Kans.), 1934 d.m.
600-705

Topeka (Kans.), 1934 d.m.
600-706

Security (Kans.), 1934, d.m.
600-707

Holton (Kans.), 1934 d.m.
600-708

Valley (Iowa), 1934 d.m.
600-709

Blair (Nebr.), 1934 d.m.
600-710

Topeka, Security Benefit Assn., tip of flag pole (Kans.), 1934 n.d.
600-711

Topeka, Security Benefit Assn., power house, conc. stack (Kans.), 1934 n.d.
600-712

Fig. 10--Sample source document from which latitude, longitude, station name, and serial number are punched manually into cards 1 and 5

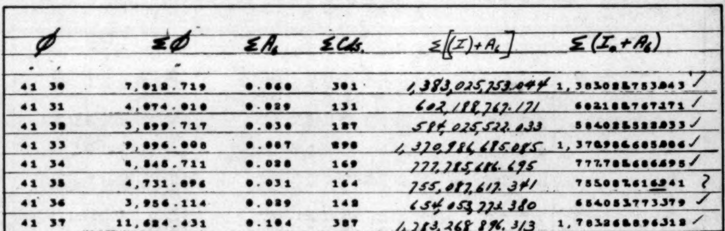
Card 1 (see Fig. 1)--After the longitude of origin (λ_0) is punched into the card, it is ready for the basic operations indicated on the card. $\Delta\lambda$ is computed on the calculating punch. It will be noted that since $p = 0.0001\Delta\lambda''$, the result of $\Delta\lambda$ is punched in two fields. The degree, minutes, and tenth position of seconds value are punched in columns 28-31 (under $\Delta\lambda$) and the unit position and the decimal portion of seconds are punched in columns 36-39 (under p); for example, $\lambda_0 = 98°$ $59' 60''$; $\lambda = 96° 08' 09.858''$; $\Delta\lambda = 2° 51' 5''$, which is punched in columns 28-31; p = 0142, punched in columns 36-39.

The conversion of $\Delta\lambda$ into seconds is performed by sorting the detail cards with conversion masters and gang punching result into columns 32-35 (under p). A_6 is obtained by sorting (with A_6 Master Cards) on columns 1-2 and gang punching. Computations of p^2, p^3, p^4, and p^5 are self explanatory.

Card 2 (see Fig. 4)--Pertinent information, latitude, p^2, p^4, A_6, serial number, and source are transferred from Card 1 onto Card 2. During this operation, the card number (2) is gang punched into the card in column 80. The new Card 2 is sorted in ascending sequence on degrees and minutes of latitude (columns 1-4). These are next merged on the collator with master cards 4A (Fig. 3). After the master cards are merged with the detail cards, Cards 2 are in groups of the same degree and minutes of latitude with the master card preceding the detail Card 2. The cards are now ready for interpolation. Values (I) + A_6, (II), and p^4(III) are computed by use of the master card and then N, the northing, is computed after the master cards are removed.

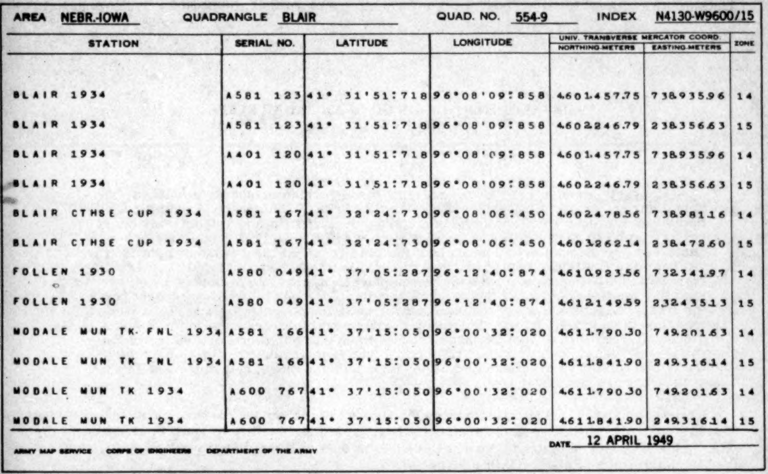
Card 3 (see Fig. 6)--Pertinent information, latitude, p, p^3, p^5, serial number, as well as source are transferred from Card 1 onto Card 3. The card number (3) is gang punched during this operation. Card 3 is sorted next on columns 6, 5, 2, and 1 in preparation for punching the correction of Δ^2IV from master cards. The cards are then sorted in ascending sequence on latitude in degrees and minutes (columns 1-4). The detail Cards 3 are then merged on the collator with master cards 4B (Fig. 5). These are now in groups according to the degree and minutes of latitude with the master card preceding the detail cards. The cards are now ready for interpolation. The values of (IV), (V), and p^5(VI) are computed by using the master cards 4B. Then p^3(V) + p^5(VI) and finally E, the easting, are computed.

All operations performed on the calculating punches were either checked on another calculating punch according to standard procedure or the tabulator was used for summation. When master cards were used, it was found advantageous to use the tabulator to sum pertinent factors from the detail cards and then perform a manual computation. In Figure 11 is a tabulation showing how (I) + A_6 is checked. If the added totals do not agree with the manual calculation, each detail card falling in the error group is checked on the calculating punch and errors isolated using standard procedure. In Figure 8 the values under latitude 41° 35′ do not check. This group is checked on



ϕ	$\Sigma\phi$	ΣA_6	Σ Cds.	$\Sigma[(I)+A_6]$	$\Sigma(I_0+A_6)$
° ′	″				
41 30	7,012.719	0.060	301	1,383,025,753.044	1,383025753043 ✓
41 31	4,074.010	0.029	131	602,188,767.171	602188767171 ✓
41 32	3,899.717	0.030	127	584,025,522.033	584025522033 ✓
41 33	9,896.008	0.087	298	1,370,986,685.085	1,370986685086 ✓
41 34	5,848.711	0.028	169	777,785,686.695	777785686695 ✓
41 35	4,731.896	0.031	164	755,087,617.341	755087616941 ?
41 36	3,956.114	0.029	142	654,053,772.380	654053773379 ✓
41 37	11,624.431	0.104	387	1,783,268,896.313	1,783268896312 ✓
41 38	5,093.873	0.053	100	829,750,806.481	829750806481 ✓
41 39	6,770.358	0.049	202	931,571,063.964	931571063963 ✓
41 40	6,681.588	0.059	229	1,056,481,072.314	1,056481072314 ✓
41 41	8,077.621	0.086	324	1,495,316,331.343	1,495316331342 ✓

Fig. 11--Summation checking; latitude ϕ, degrees and minutes are group indicated; totals obtained of seconds for latitude ϕ, A_6, detail cards in group, and interpolated values of (I) + A_6 from Card 2; values are checked against manual computation using U. T. M. Tables; note error appearing in group of ϕ = 41° 35′; errors in this group are isolated in calculating punch by standard procedure



AREA NEBR.-IOWA QUADRANGLE BLAIR QUAD. NO. 554-9 INDEX N4130-W9600/15

STATION	SERIAL NO.	LATITUDE	LONGITUDE	UNIV. TRANSVERSE MERCATOR COORD.		ZONE
				NORTHING-METERS	EASTING-METERS	
BLAIR 1934	A581 123	41° 31'51".718	96°08'09".858	4601.457.75	738.935.96	14
BLAIR 1934	A581 123	41° 31'51".718	96°08'09".858	4602.246.79	238.356.63	15
BLAIR 1934	A401 120	41° 31'51".718	96°08'09".858	4601.457.75	738.935.96	14
BLAIR 1934	A401 120	41° 31'51".718	96°08'09".858	4602.246.79	238.356.63	15
BLAIR CTHSE CUP 1934	A581 167	41° 32'24".730	96°08'06".450	4602.478.56	738.981.16	14
BLAIR CTHSE CUP 1934	A581 167	41° 32'24".730	96°08'06".450	4603.262.14	238.472.60	15
FOLLEN 1930	A580 049	41° 37'05".287	96°12'40".874	4610.923.56	732.341.97	14
FOLLEN 1930	A580 049	41° 37'05".287	96°12'40".874	4612.149.59	232.435.13	15
MODALE MUN TK FNL 1934	A581 166	41° 37'15".050	96°00'32".020	4611.790.30	749.201.63	14
MODALE MUN TK FNL 1934	A581 166	41° 37'15".050	96°00'32".020	4611.841.90	249.316.14	15
MODALE MUN TK 1934	A600 767	41° 37'15".050	96°00'32".020	4611.790.30	749.201.63	14
MODALE MUN TK 1934	A600 767	41° 37'15".050	96°00'32".020	4611.841.90	249.316.14	15

DATE 12 APRIL 1949

ARMY MAP SERVICE CORPS OF ENGINEERS DEPARTMENT OF THE ARMY

Fig. 12--Final listing of results within 15-minute quadrangle in ascending sequence of latitude

the calculating punch according to standard procedure and errors isolated.

Card 5 (see Fig. 2)--This is the final card showing the original values punched manually and the computed Universal Transverse Mercator Coordinates transferred from Cards 2 and 3. The final cards are sorted according to ascending latitude within fifteen-minute quadrangles. Figure 12 shows a final listing of results.

Conclusion--There are approximately 650,000 geographic positions within the United States of which approximately seven per cent are duplicates and 17 per cent fall in overlapping areas. This gives a total of over 750,000 conversions which are being made at the rate of 30,000 per month.

Computations are now in progress to convert coordinates in Europe to Universal Transverse Mercator Coordinates. It is planned to convert well over 1,500,000 values in foreign areas. The conversions will be performed by the use of complex numbers expanded to a series which may involve the fifth order. Since the conversions involve the transformation directly from local grid systems to Universal Transverse Mercator Coordinates on a local spheroid, the computations are more involved than converting from geographic positions to Universal Transverse Mercator Coordinates.

Army Map Service,
Washington 25, D. C.

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Transactions, American Geophysical Union Volume 31, Number 4, August 1950

DEVELOPMENT OF AN OPTICAL RADAR SYSTEM FOR SURVEYING PURPOSES

Charles R. Hoover

Abstract--An electro-optical surveying system which measures distances, horizontal angles, and vertical angles directly is described. The system consists of an optical transmitter which emits microsecond pulses of light and a photosensitive receiver combined to use different annular sections of the same parabolic reflector. Distances are determined by measuring the time required for a pulse of light to travel from the position occupied by the equipment to another station where a retro-reflector is positioned, and back to the equipment. Angles are measured by so restricting the field of view of the receiver portion of the optics that return light from the retro-reflector cannot enter the receiver until an accurate pointing of the equipment has been made. An existing piece of equipment is described which measures distances from zero to five thousand yards with an estimated probable error of two yards, and measures horizontal and vertical angles to an accuracy of plus or minus thirty seconds. The practical limitations on maximum range, and range and angular accuracies are discussed.

Introduction--The instrument described is capable of measuring distances and angles directly. While its ability to measure distance directly lends speed and facility to many operations, in general the accuracy of distance measurements is at best of third-order quality. Angles are measured to an accuracy of plus or minus 30 sec.

This device determines distance by measuring the time lag between the emission from the instrument of a pulse of light (approximately one microsecond duration) directed toward a reflector at the distant station, and its return to the instrument. Since the transit time is proportional to the distance traveled by the light pulse, the circuit measuring the time interval is readily graduated in yards. Angles are measured on conventional horizontal and vertical circles, sensitivity in pointing being achieved by accepting the returning pulse of light from a very restricted field of "view."

Description of the instrument--Before the characteristics of the instrument as they apply to surveying can be discussed in detail it is necessary to consider the operating principles more completely. Figure 1 is a photograph of the gap-lamp which is the source of the outgoing pulse of light. The light is produced by a spark between the electrodes, caused by the discharge of a condenser which is brought slowly to the breakdown voltage of the lamp. The spark thus produced has a high surface brightness and a duration of about one microsecond. The flashes recur at a rate of about 20 per sec.

Figure 2 presents a simplified block diagram of the electronic portion of the device. The electrical disturbance caused by the initiation of the spark in the lamp is used to start the sequence of events in the electronic system which ultimately compares the time interval until the return of the pulse of light from the target reflector against an internal, free-running oscillator which serves as a time base. The flash of the source starts the upper trace on the cathode-ray tube by triggering the sweep generator. The return light from the distant station strikes the photomultiplier which is in the same optical system with the flash-lamp and the resultant electrical pulse when amplified by the preamplifier and the vertical amplifier causes the echo pip on the upper sweep. The length of the sweep between the left side and the echo pip is a function of the time interval required for the light to travel to the reflector and return. The problem is to measure the time interval between the inception of the sweep and the instant when the echo pip was traced on the screen. To do this another sweep showing timing pulses from the internal oscillator is presented below the first sweep. Because the instant of the lamp's flashing bears no fixed phase relationship to the free-running oscillator it is now necessary to compare a time interval of the same duration, which does have a fixed phase relationship to the free-running oscillator, against the first interval. Therefore, at the same instant the first sweep starts, a pulse is fed to the second sweep delay circuit which prepares to again trigger the sweep generator a few microseconds after completion of the first sweep. The exact instant at which the sweep generator is tripped the second time is determined by the oscillator.

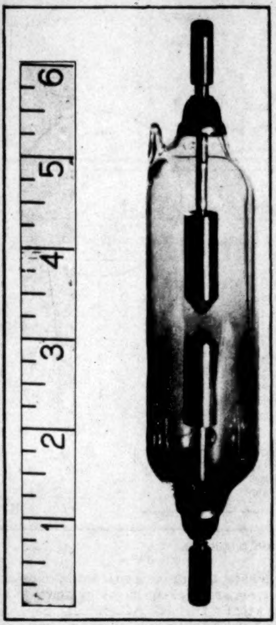

Fig. 1--Pulsed light source

The second sweep delay circuit synchronizes its trigger pulse, represented on the block diagram by the dotted line to the sweep generator, with the occurrence of a peak on the wave train generated by the oscillator. The second sweep is then presented on the cathode-ray tube slightly below the first sweep, The pips on the lower sweep are formed by the range pip generator, one for each cycle of the oscillator output. The position of these pips along the sweep is determined by the phase shifter which feeds a sine wave into the range pip generator of the same frequency as the oscillator but with a phase relationship to the oscillator which can be varied continuously from a phase angle of 0° to 360°. The gate circuit serves to keep the range pips from appearing on the upper sweep. Since this cycle of events recurs approximately 20 times a second, the pattern appears constant. By turning the dial on the phase shifter, the range pips appear to move across the lower sweep, and one of them can be made to line up with the echo pip on the upper sweep, peak to peak. Although several pips appear on the lower trace it requires a complete turn of the phase shifter dial to move from one range pip to the next so the dial reading will be the same regardless of which range pip is moved into line with the echo pip. On the upper sweep, then, the position of the pip is a function of the distance between the instrument and the station where the reflector is positioned, while on the lower sweep the pip position is a function of the phase shifter dial position. Obviously the phase shifter dial can be calibrated to read distance and will indicate the range of the target, when the two pips are made to coincide.

In many conventional radar systems, a free-running oscillator which serves as the time base initiates the outgoing pulse, thereby establishing the instant of zero time at some specific phase of the oscillator. At first thought it would seem that for simplicity the flash source should be triggered in the same manner and that this trigger pulse should be used to start the sequence of events. It has been found, however, that a variable time lag between the application of a trigger pulse to the lamp and the instant of peak light emission occurs when the lamp is triggered. This would represent a random range error and it is for this reason that the lamp is allowed to flash spontaneously and start the timing cycle from there.

The oscillator frequency is so chosen that the distance between two pips on the lower trace represents a range of 500 yd. In order to magnify a 500-yd interval to gain precision in matching the pips but at the same time be able to extend the range of the instrument beyond 500 yd, a time interval representing 1000 to 1500 yd is expanded and occupies most of the sweep, while all the balance of the time interval representing the entire range of the instrument is crowded into the spots on each end of each sweep (see Fig. 3). The coarse range switch (see Fig. 4) selects the portion of the range expanded on the screen to bring the echo pip approximately to the center of the screen thereby determining the range to the nearest 500 yd, while the fine range dial moves the range pips to match the position of the echo pip, thereby determining the position of the echo pip within a given 500-yd range. The sum of the two dial readings is the range in question. The distance read from the dial is, of course, line-of-sight distance.

Possible sources of error--Errors in the range determination are introduced in the following ways: deviation of the oscillator frequency from 327.76 kc/sec (one cycle of which occurs in the time required for light to travel 500 yd and return); operator error in setting the peaks of a range pip and the echo pip into coincidence; errors in the zero setting; errors in the calibration of the phase shifter; and variations in sweep speed bwteeen the upper and lower sweeps. The oscillator is crystal-controlled and temperature-compensated providing frequency stability good to approximately one part in 20,000. Error in the oscillator frequency of this magnitude will introduce a range error of approximately one part in 20,000 in each 500-yd interval. This is less than one inch

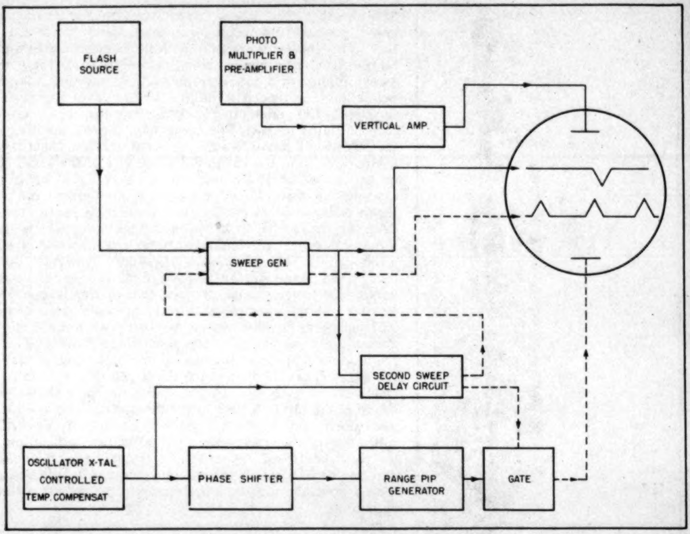
FLASH SOURCE
PHOTO MULTIPLIER & PRE-AMPLIFIER
VERTICAL AMP.
SWEEP GEN.
SECOND SWEEP DELAY CIRCUIT
OSCILLATOR X-TAL CONTROLLED TEMP. COMPENSAT
PHASE SHIFTER
RANGE PIP GENERATOR
GATE

Fig. 2--Block diagram of electronic system

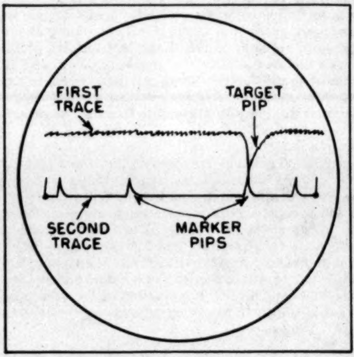
FIRST TRACE
TARGET PIP
SECOND TRACE
MARKER PIPS

Fig. 3--Appearance of scope when pips are matched

for each 500 yd and is insignificant in comparison with other errors present in the system. It has been found that a trained operator can reproduce a setting of the fine range dial for a given range to within half a yard of some arbitrary value. (What this value depends on the zero setting of the instrument and the particular operator's method of matching pips.) This is merely an indication of the precision with which the pips can be matched. The spread of settings for a given range will be considerably increased if the echo pip is weak or unsteady. Unsteadiness will be caused by refraction of the air under the same conditions and to the same extent that this effect is noticeable in a telescope. There is an additional unsteadiness due to differences in the brightness of successive flashes of the source.

These conditions do not move the position of the echo pip along the sweep, but they cause variations in the pip height. Since the operator is matching the pips, peak to peak, this matching becomes more difficult. At night, with ranges up to 5000 yd and reasonably clear atmosphere the unsteadiness is not appreciable. In general, the probable error involved in a single range setting is estimated to be two yards. The zero range setting is made by placing a reflector directly in front of the instrument, matching the pips, and then setting the dial to zero or some other known value. The error in making the zero setting should be no greater than the operator's error in matching pips under ideal conditions, or plus or minus one-half yard. The phase shifter introduces an error that varies throughout one revolution by about

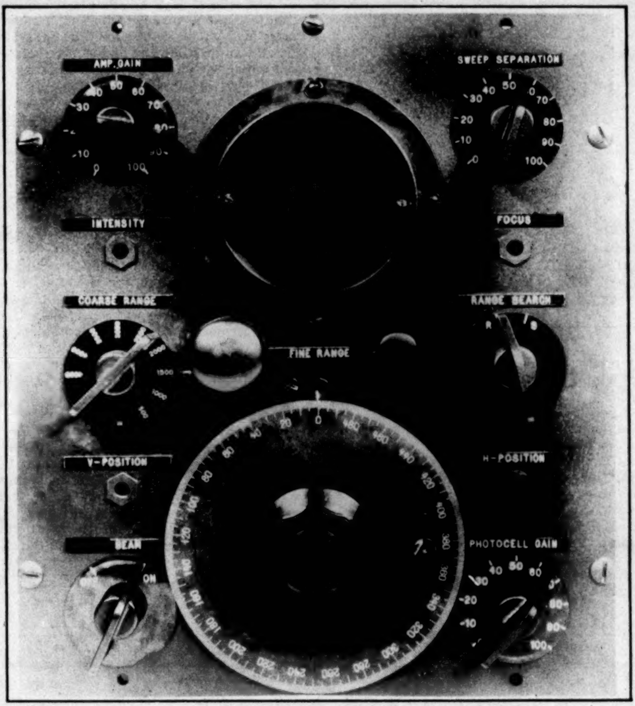

Fig. 4--Electronic control panel

± 1°, or plus or minus 1/360 of a 500-yd interval, or plus or minus 1.3 yd. This is a cyclic error that repeats once per revolution of the range dial. It results from harmonics in the output of the oscillator, asymmetry in the bridge, which is part of the phase shift circuit, and asymmetry in the mechanical construction of the phase shifter. This error can be partially averaged out by introducing four different zero settings spaced in quadrature around the fine range dial and making a range measurement with each setting. In other words, the phase shifter dial would be set at zero when making the zero setting for the first determination, then at 125 yd when making the zero setting for the second determination, third at 250 yd, and finally at 375 yd. The phase shifter error is the largest of all the errors which a single range determination should introduce, but by taking a set of readings as described above, this error can be reduced essentially to the operator error in matching pips. The variation in sweep speeds between the upper and lower sweeps appears to introduce errors no larger than the oscillator errors and hence can be ignored. From this discussion of the range errors, it can be seen that the largest error, error in matching the pips, is one which is independent of distance, hence the range accuracy cannot be expressed on a percentage basis. On a percentage basis, the accuracy improves with distance and this should continue, if the range of the system were extended, until the oscillator error became the predominant error. The system could be extended considerably in range by increasing the size of the instrument's optics or the number of reflectors at the target, or both.

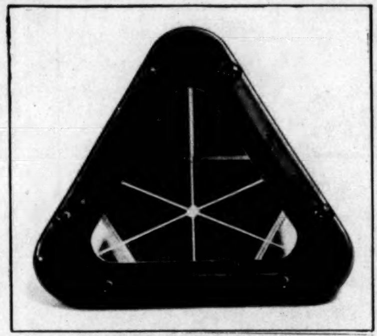

Fig. 5--Retro-reflector

The optical system--Before consideration of the angular accuracy of the instrument can be made, the optical system must be described. The station to which the distance is being measured must be equipped with a reflector. There are diffuse reflections from many objects within the field of view of the equipment but by placing a reflector at the particular position to which the range is to be measured the intensity of the light from this point is made much greater than the diffuse reflection from other objects, thereby less intense diffuse reflections are over-ridden, and the particular point to which the range is to be measured is identified. It is possible to get a reflection from a highly reflective, properly oriented object other than the target but the chance of this is slight. Under circumstances for which the chance of a false reflection is great, communication between the instrument and the distant station will easily establish whether or not the instrument is pointed at the desired target. The most effective reflector seems to be a corner cube reflector of the type shown in Figure 5. This type of target has the characteristic of returning the light falling upon it back over the same optical path by which it came.

The optimum orientation is for the optical axis of the reflector to be pointed toward the source, but the orientation of the reflector may vary as much as 15° from this optimum in any direction and still return a satisfactory amount of the light falling on it. Since this type of reflector is retrodirective, it is necessary that the transmitting and receiving optical systems of the instrument be as close to each other as possible. To achieve this a coaxial arrangement is used as shown diagrammatically in Figure 6. AG is a parabolic reflector divided into two annular sections. The center portion, CD, is not used. The outer ring, AB-EG, serves as the transmitting section. The flash-lamp F occupies the focal point of this parabola. The plane mirror, ST, shades the section

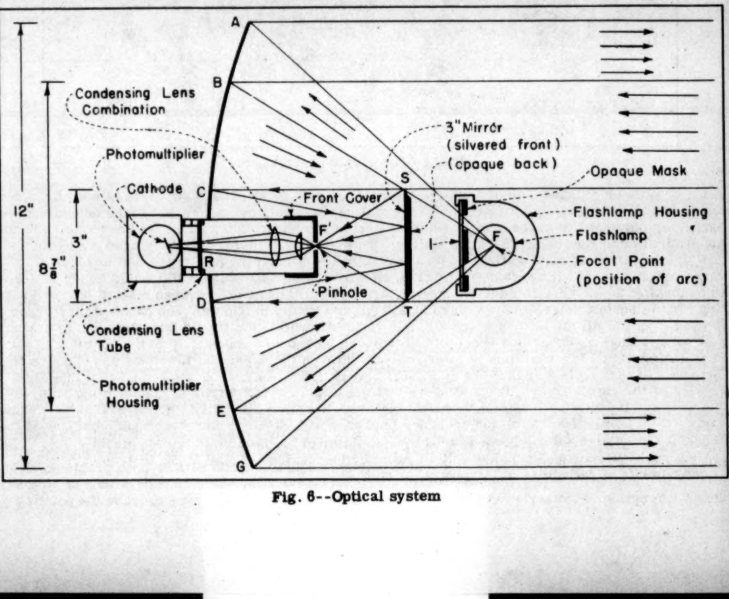
A
B
C
D
E
G
S
T
F
F'
R
I
12"
8 7/8"
3"
Condensing Lens Combination
Photomultiplier
Cathode
Front Cover
Pinhole
3" Mirror (silvered front) (opaque back)
Opaque Mask
Flashlamp Housing
Flashlamp
Focal Point (position of arc)
Condensing Lens Tube
Photomultiplier Housing

Fig. 6--Optical system

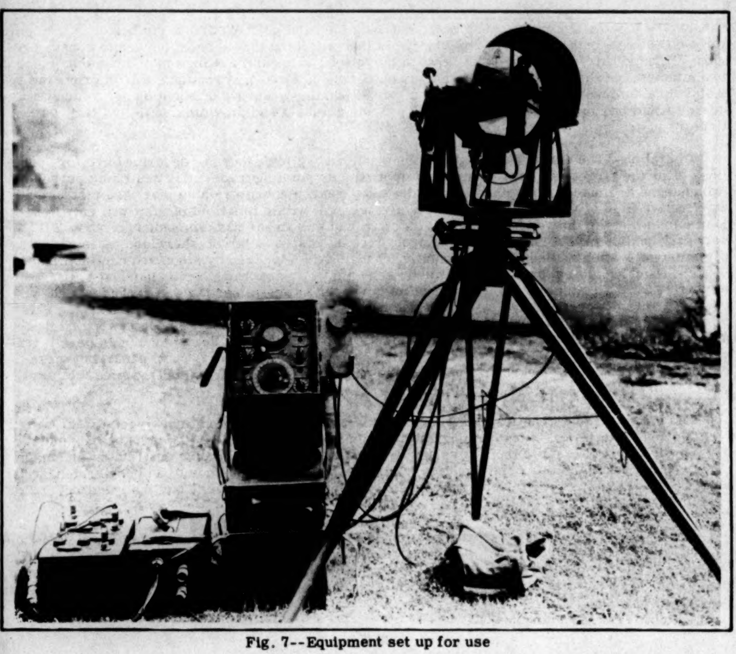

Fig. 7--Equipment set up for use

BE of the parabola from the source and prevents radiation from going directly from the source to the receiver system. The problem of keeping outgoing radiation from striking supports and other objects in the optical head and thus scattering directly back into the receiver system is severe. In fact, it puts a lower limit on the range which can be measured. There will be a pip on the beginning of the sweep due to the return of scattered light from within the head and unless the target is 40 to 50 yd away the return from it will merge with the pip due to scatter giving a single broad pip with a maximum which is a compromise between the true range and zero.

The return radiation is collected by the ring BC-DE. The width of this ring must be approximately the largest dimension of the retro-reflector used as a target. For this reason, if it is desirable to obtain a stronger return from the distant station than a single reflector such as shown in Figure 5 provides, several reflectors of the same dimension must be used, rather than to use a larger single retro-reflector. From the receiving ring of the parabola the light is folded back by the plane mirror, ST, to another focal point F′. From there the return light goes through a lens system and is spread out somewhat to more nearly cover the area of the cathode in the photomultiplier. The pinhole at F′ serves the same function in this system as the reticle in a telescope. Its position must be so adjusted as to cause the line of sight of the instrument to intersect the horizontal and vertical axes and be perpendicular to the horizontal axis. The limitation of pointing accuracy rests almost entirely in the precision of the parabola and the size of the pinhole. The image of the target reflector an appreciable distance away is essentially a point and if the circle of confusion of the parabola is small, the pinhole may have such dimensions as to limit the instrument's field of view to very little more than the target itself. The model tested was equipped with 30-sec circles and it was found that repeated pointings could be made with as good precision as the circle could be read provided the return signal was good. The same considerations of atmospheric refraction and other weak signal effects reduce the angular accuracy because the pointing is made by moving the optical head while watching the echo pip. Maximum pip height is the operator's objective. Unfortunately the pip height is the vary thing that is changing due to causes previously described. When the signal is weak the spread of angular settings may include as much as two minutes, but the spread is random and an average of a group of readings will improve the results. It is believed that the optics have an actual pointing accuracy of about 20 sec. Angular accuracy could be improved by using a more precise parabola and providing sufficient target reflectors to assure a strong return signal.

Operation--The arrangements, both electronic and optical, thus far described are for conditions after the target has been located and approximate pointing made. For searching purposes two changes are made. The pinhole is replaced by a large aperture which gives the instrument an effective field of view of about 2°. Actually, a target within this field of view will only be illuminated when the image of the spark falls on it, and this image has dimensions of about 1.1° vertically and 20 min horizontally. The procedure in locating a target, therefore, is to set an approximate elevation and search horizontally, changing the elevation by steps and continuing the search, watching the cathode-ray tube for an echo pip. For searching, it is also best to present the entire 5000-yd range of the instrument on the upper sweep rather than an expanded 1000-yd portion. This is accomplished by turning the range-search switch (see Fig. 4) to search. This position eliminates the lower sweep on the cathode-ray tube and slows the upper sweep so that any reflection within the range of the insturment will be presented. Remote controls permit traversing and elevating the optical head (see Fig. 7) from the electronic control panel, but these controls can be used for searching only and the tangent screws on the tripod-mounted unit are used for final pointing.

The piece of equipment described operates from two six-volt automotive-type storage batteries for a period of about six hours continuously. Exclusive of the batteries it weighs approximately 200 pounds and breaks down into six packages. One man can assemble the instrument and make the first range and azimuth determination in about 15 min. The equipment can be operated day or night but has better accuracy at night because of the better signal quality and the resultant steadier pip height.

Acknowledgments--Acknowledgment is made to the Armour Research Foundation which designed and built the unit described under the sponsorship and supervision of the Engineer Research and Development Laboratories; and to the Bell Telephone Laboratories for their wartime work under NDRC contract resulting in basic developments in range measuring equipment utilizing the transit time of light pulses.

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Transactions, American Geophysical Union Volume 31, Number 4 August 1950

AN INVESTIGATION OF TWO-SECOND MICROSEISMS ASSOCIATED WITH COLD FRONTS AND A NEW METHOD FOR TRACKING THE COLD FRONT CENTER

J. Joseph Lynch

Abstract--Two tripartite stations have been set up six miles apart. Each consists of three two-second horizontal seismographs recording at a drum speed of 600 mm/min.

In addition, three stations equipped with two-second vertical and horizontal instruments have been set up on a triangle with sides of 8.3, 6.75 and 6.0 miles.

Results to date are somewhat confusing. There is evidence of a microseismic source southwest of New York. There is also evidence of wave trains identifiable between two stations six miles apart indicating a velocity of microseismic waves of less than one mile per second. A new method is presented of recording microseisms on an oscilloscope and reading the time differences of each pair of stations along the axis of the observed Lissajous ellipse.

Constant observation since July 1947 has shown conclusively that two-second microseisms are associated with the passage of a cold front. Their periods vary from 1.5 to 3.0 sec, the period often changing markedly during the passage of such a front.

As a first step in the determination of the nature and mechanism of these two-second microseisms, a tripartite station was set up on the campus of Fordham University. The left-hand diagram of Figure 1 shows the orientation of the three units. A two-second horizontal Sprengnether instrument oriented east-west was set in a concrete box at each station and wires led thence to the main vault where the recording is done. At first a triple drum with a speed of 300 mm/min was used but this was found to be too slow. A speed of 600 mm/min is now used and a two-meter lens replaced the regular one meter lens on the galvanometer to give greater magnification. The instruments are connected to the central recording room by two conductor shielded microphone cable. At first the cable was laid underground, but rats and ground currents forced an overhead route. Here lightning has given trouble, burning out one seismometer coil and one galvanometer coil, but on the whole this arrangement has proved satisfactory. Gilmore's method of reading the records has been used and found very helpful. A slight modification has been introduced, namely that important records are first transferred to autopositive film to facilitate their reading. The records are superimposed in pairs on an illuminated glass covered box and time differences read by a traveling microscope or binocular microscope with eyepiece scale. Fairly consistent results have been secured from the various records but a larger time scale would be highly desirable and is being considered.

Having found the tripartite station satisfactory for determining direction, the next step in the investigation was to establish a second tripartite station so that the intersection of the two directions could be used to establish the source of the wave disturbance. This second station was set up at the New York State Maritime College at Fort Schuyler in the East Bronx. The center diagram in Figure 1 shows the orientation of these units. One unit is housed in the basement of a dormitory, one in a disused well and the third is placed on a concrete embankment near the water front and enclosed in a steel shell which is bolted to the ground. The triangle is somewhat too flat but the attempt to widen it by putting the third instrument further out, into the water on a concrete pier proved futile. The propeller patterns of every passing tug and launch completely masked the microseisms. Pending the arrival of milder weather for a rearranging of this tripartite station it was decided to set up a vertical and horizontal two-second instrument at Fort Schuyler and also at Iona College - these two forming with Fordham a triangle with sides of 8.3, 6.75, and 6.0 mi, oriented as shown in the right-hand diagram of Figure 1. Standard Electric time clocks checked by WWV signals were installed at both places and have now been in operation sicne early April. The vertical instruments were all operated side by side in the main vault at Fordham before being set up at the outlying stations. All gave identical records. The horizontal instrument now at Fort Schuyler was similarly checked against the horizontal instrument of the Fordham Station. The Iona horizontal instrument has not been set up, pending the outcome of results observed on the three vertical instruments.

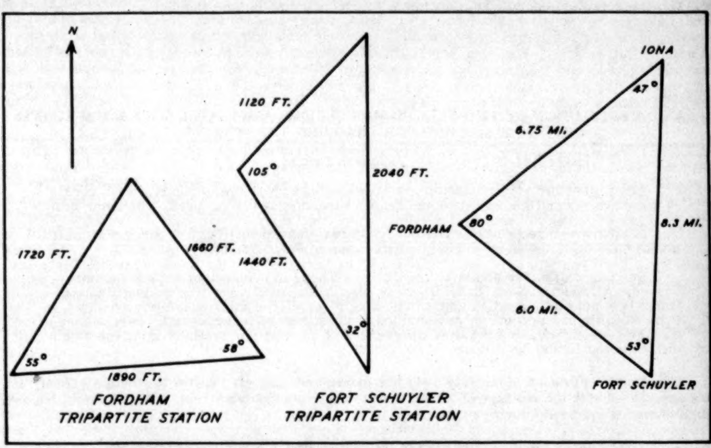
N
1720 FT.
1660 FT.
1890 FT.
55°
58°
FORDHAM
TRIPARTITE STATION
1120 FT.
105°
2040 FT.
1440 FT.
32°
FORT SCHUYLER
TRIPARTITE STATION
IONA
47°
6.75 MI.
FORDHAM
80°
8.3 MI.
6.0 MI.
53°
FORT SCHUYLER

Fig. 1--Orientation of Fordham and Fort Schuyler tripartite stations, and of those stations with the Iona seismic stations

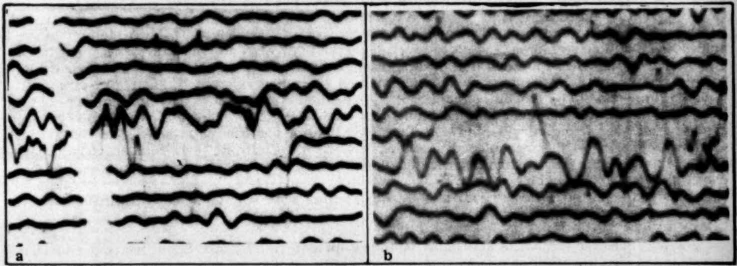

Fig. 2--Hokkaido earthquake, February 28, 1950; (a) Iona record; (b) Fort Schuyler record

The instruments were in operation but without accurate time checks at the time the earthquake of February 28, 1950 occurred north of Hokkaido, Japan. While the main phases of the quake were remarkably similar on all records, there were some minor differences that need investigation. The first part of the vertical records of the two outlying stations is reproduced here in Figure 2. The second lines (15 min later) show agreement in the main but with many points of difference. These two stations are 8.3 mi apart and are in a direct north-south line; the earthquake come from the northwest. The third record is not shown since it was out of phase at the time and would only confuse the picture.

It was decided that the first problem was whether or not the same wave train could be identified conclusively at stations so far apart. Several cold fronts have passed since the stations have been in operation but they have usually been associated with storms from other directions.

The most striking evidence that wave trains can be so identified came on March 4, 1950. Previously an occasional similarity had been noticed but all such cases were considered doubtful. The records of March 4 seemed to leave little room for doubt. When the records for Fort Schuyler and

Fordham were superimposed and compared they were so alike around 00^h15^m that for a while the writer thought he was superimposing two records of the same station. The consensus of four observers here was that it was definitely the same wave train that was being recorded at the two station six miles apart. However, it was deemed advisable to send copies of the reocrds to Beno Gutenberg and James B. Macelwane for comment. Gutenberg wrote that he and Benioff both felt that many of the wave trains were the same, but that the direction seemed to change, a phenomenon they had noticed in their vector method of recording. Macelwane and Blum also wrote that many of the wave trains seemed to be the same but that the conclusion to be deduced was doubtful.

Assuming on the basis of this confirmation that they were the same wave trains and using Gilmore's formula for tripartite determinations, the center of the disturbance was shown to be from a direction S 16° E. The predominant direction of the microseismic source around that time was from the southwest as determined from our local tripartite station on the campus. There was no record from the tripartite group for the particular wave train studied since this tripartite group only operates five minutes every half hour. Search on the tripartite group for a source from the southeast within a few hours of the time of the wave train studied (00^h45^m, March 4, 1950) failed to disclose any. Many of the wave trains, which we now presume the same, were definitely Rayleigh waves since they appear both on the vertical and horizontal instruments. One group definitely indicates direction from the west, that is, the motion is east-up-down-west. This agrees with the direction determined by the tripartite station. The particular wave train measured to get the direction S 16° E consisted of a single peak on the three verticals and hence no direction could be ascertained by comparison with the horizontal instruments.

The following serious difficulties at once present themselves: (1) The existance of a source of microseisms to the southwest of Fordham seems to indicate a center on land unless of course we put the center as far away as the Gulf of Mexico. (2) The velocity of these microseismic waves on the assumption that we are dealing with the same wave train would be extremely low. The distance between Fort Schuyler and Fordham is six miles and the time interval between the two stations is

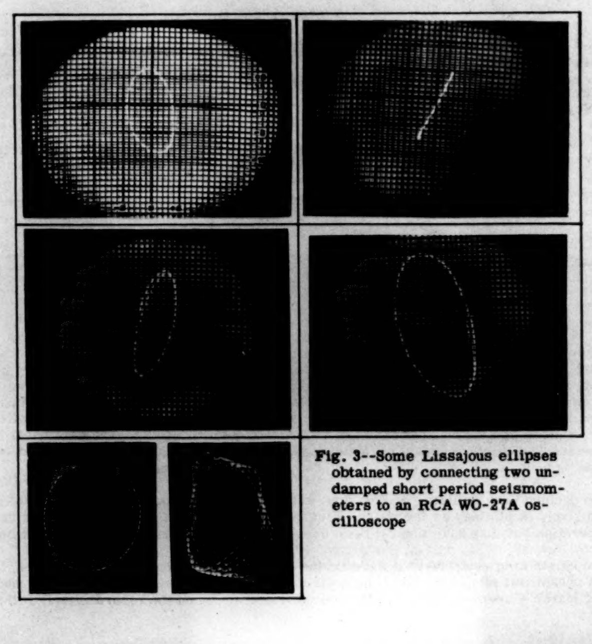

Fig. 3--Some Lissajous ellipses obtained by connecting two undamped short period seismometers to an RCA WO-27A oscilloscope

around nine seconds. This would imply a velocity of about 2/3 mile per second which is extremely low. (3) One would expect some indication from the Fordham Tripartite Station of a source to the southeast if the wave train singled out really had its source in that direction. (4) Two more recent cold fronts which gave good records at all three stations failed to show wave trains readily identifiable as the same at the three stations.

Summarizing all the evidence accumulated from the reading of dozens of sets of records (by four or five different observers) results in the following observations: (1) During any one storm, sources from widely differing directions are indicated by the tripartite station records. (2) There seems to be a definite source of some of these microseisms to the southwest of New York City. (3) The writer feels convinced that the same wave trains in the storm of the morning of March 4, 1950 were identified at New York and Fort Schuyler (six miles apart) but they were not so readily identifiable at the third station, Iona, 8.3 miles from Fort Schuyler.

Hence it has been decided to find a fourth location lying nearer to Fordham than the other two stations in an effort to find out with more assurance how far such a wave train can travel without losing its identity.

Meanwhile, to make it easier to read time differences on records of the tripartite group a method is being worked out whereby the seismometer output of each pair of stations in turn is thrown on an oscilloscope and the phase difference read directly along the axis of the Lissajous ellipse. Some of the ellipses obtained are shown on Figure 3. Since the periods and amplitudes of the tripartite stations are the same the phase difference is a function of the eccentricity of the ellipse [TERMAN, 1935]. The chief difficulty has been to obtain a DC amplifier that will respond sufficiently well to the extremely low voltage of the seismometer output. The Liston-Folb DC amplifier developed in the laboratories of General Motors seems to satisfy the required conditions but up to the time of writing we were still awaiting its arrival for testing.

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AN ELECTRONIC VIBRATION METER

Edward F. Carome

Abstract--A completely portable electronic instrument which appears to have wide application in the field of vibration measurements has been developed. The circuit is such that a meter responds to the maximum amplitude of a transient or continuous voltage input. The indication is held for an indefinite time. Used in conjunction with a commercial vibration pickup, this instrument provides a simple and convenient means of measuring accelerations in structures and of referring them to a standard index of damage.

One of the most important fields of applied geophysics is the measurement of structural vibrations. Whether these vibrations are caused by machinery, traffic, blasting, or other sources, their analysis and control claim a large share of the effort of the geophysical engineer.

Several methods of vibration analysis are in current use. All employ some type of pickup, essentially a seismograph, whose motion may be recorded directly on photographic paper. A more common method uses the pickup as a transducer, changing mechanical force into electromotive force. In this system, the output of the pickup is amplified and recorded on photographic paper or with pen and ink. A variation of this method uses a meter to indicate the amplitude of the output of the pickup.

When the measured vibrations are continuous, the permanent record of the oscillograph gives both the amplitude and the wave-form of the motion. On the other hand, the portability and simplicity of operation of the meter type of indicator often far outweigh all other factors. The disadvantage of the meter indicators currently available is that they are unsuited to record a single sharp pulse or transient vibration. It is the purpose of this paper to describe a meter type of indicator which can be used to measure the peak amplitude of a transient voltage and to show how this indicator is applied to vibration analysis.

Basically, this instrument is a pulse voltmeter and, as such, it can be used in conjunction with different types of transducers to measure the peak displacement, velocity, or acceleration of a transient vibration. However, for the sake of clarity, its specific application for the measurement of acceleration will be treated most fully in this paper.

As an accelerometer, this instrument consists of three basic parts: an inertia type pickup, an amplifier, and a pulse recording circuit. Each of these elements has been designed or incorporated in this instrument with the intention of simplifying its operation and increasing its portability.

The inertia pickup is a piezo-electric transducer which has been designed especially for use with this instrument by the Brush Development Company. Unlike many pickups of this type, its sensitivity does not vary with changes in temperature. Electrically it fits the instrument perfectly since over a wide band of frequencies the voltage output of this pickup is directly porportional to the acceleration. Physically it is rugged, small, and light, weighing less than one pound.

The second part of this instrument is a conventional amplifier with a voltage gain of about six hundred. This circuit employs two sub-miniature triode tubes, used primarily because of their small size and extremely low filament and plate current.

The final element, the pulse recording circuit, can be broken down into two basic parts. The first is a capacitor that charges to the peak voltage of a single pulse. This capacitor is capable of retaining its charge for an indefinite period of time. The second part is a special type of voltmeter which utilizes an ordinary panel meter movement as a visual indicator. It is used to measure the voltage across the capacitor without in any way affecting its charge.

The actual operating cycle of the instrument is very simple. When subjected to a transient vibration the pickup generates a voltage that is proportional to the acceleration. This voltage is

amplified and then impressed across the capacitor which effectively "remembers" the peak amplitude of the voltage pulse. The meter provides a visual indication of this voltage. To give each reading the greatest practical meaning, the meter scale is calibrated in fractions of the acceleration of gravity. Since the capacitor retains its charge this reading will persist for many hours or until the operator himself discharges the capacitor with a push-button switch. To extend the range of the accelerometer there is also a three-position range switch which provides full scale meter readings of one-tenth, one and ten times the acceleration of gravity.

The applications of this instrument are wide and varied. From the author's own experience it has been found to be ideal for determining the acceleration of vibrations caused by industrial machinery and blasting. For ordnance testing, construction and mining work, in fact, wherever there are transient or steady state vibrations, this instrument can be applied to great advantage as an accelerometer.

A commercially available form of the amplifier and indicator circuits is being manufactured by the Victoreen Instrument Company of Cleveland, Ohio. This unit is small and compact, weighing only six pounds. It can be carried and operated by one man and since it is entirely battery powered it requires no external power source. With the inertia-type pickup, it forms a completely portable accelerometer. Because of its simplicity it can be used by both technical and non-technical personnel. Once the instrument has been turned on and adjusted, it needs no further attention. For instance, for a blast measurement one man could easily set up several instruments at widely separated points. Each instrument would automatically monitor the transient vibrations produced by the blast and the operator could then return to read the peak accelerations recorded by the instruments at the different points. In this way, vibration surveys could be carried out quickly and easily.

As was stated earlier in this paper, the amplifier and indicator unit can be used independently of the inertia transducer for other types of transient voltage measurements. For example, with a displacement pickup the instrument can be converted with a minimum amount of recalibration to a displacement meter.

Acknowledgments--The plans for this instrument developed when the author had the privilege of assisting Henry F. Birkenhauer, and Edward J. Walter of the Seismological Observatory of John Carroll University. Without their generous help and advice, this instrument could not have been designed. The author would like to express his appreciation to them and to the personnel of the Victoreen Instrument Company and the Brush Development Company who have aided greatly in the development of this instrument.

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ON THE PROPAGATION OF HURRICANES

Leon Sherman

Abstract--The notion of steering has been much used in tropical meteorology. It is a somewhat hazy concept. It is preferable to consider the more precisely defined convective velocity. The computed convective velocity will be found to account for little more than half the observed variability in the direction of propagation of storms.

Introduction--The steering current has been described as the dominating factor in the control of hurricane paths, including those parts corresponding to periods of potential or actual recurvature [ALGUE, 1904; CLINE, 1926; DOBERCK, 1904; RIEHL and SCHAEFFER, 1944; and others]. Indeed, in reviewing the 1948 Caribbean and southwest North Atlantic hurricanes [NAVAER, 1949] the officer in charge of the Navy Hurricane Weather Central, Miami, states: "None of the newer (theoretical hurricane forecasting techniques) proved to be any better than the time-tested persistence and isallobaric methods, with the exception that upper level steering proved useful when these winds were available." Thus "steering wind" and "steering level" are concepts receiving serious attention from many (but by no means all) of the working meteorologists faced with the actual forecast. As an example of the opposite attitude, at an important hurricane forecasting center the following definitions of these two concepts are used (communicated to author): steering wind, the wind at a level called the steering level; steering level, a level at which the wind direction coincides with the direction of propagation of an underlying storm. There is lack of agreement as to the importance of steering. This may well be due to a lack of agreement as to the meaning to be attached to the word steering. Accordingly it is preferable to avoid this term and instead discuss the convective velocity, a concept closely akin to steering (and in the eyes of many, identical in meaning).

The propagation of disturbances in fluids--The velocity of propagation of a fluid disturbance may be considered to consist of a convective part and a dynamic part [STEWART, 1945]. For example, for a small arc length of a water ripple (see point P of Fig. 1) the convective part of the velocity of propagation is the current of the river. This portion of the velocity would be present even in the absence of any asymmetric distribution of the horizontal divergence. The dynamic portion of the velocity of propagation is that part which would appear should the stream velocity be zero. In our example, this is the radial velocity the ripple would have in still water. This part of the velocity of propagation clearly depends upon the existence of asymmetry in the horizontal divergence field, with convergence and the consequent piling up of water ahead of the ripple and the opposite behind it.

Before continuing, let us consider another example: the extratropical cyclone. It is well known that a low pressure system made up of closed circular isobars at all levels should be expected to move toward the west [BJERKNES and HOLMBOE, 1944]. Let us designate the velocity field of such a system by the symbol $\mathbb{W}_o$ and the corresponding surface pressure tendency field by $(\partial p/\partial t)_o$. Now consider a field $\mathbb{W}_o + U(z)\mathbf{i}$, where $U(z)\mathbf{i}$ is a west wind whose speed increases with elevation. This approximates a real extratropical cyclone. Its surface pressure tendency field is readily computed as

$$(\partial p/\partial t)(x, y, o, t) = -\int_o^\infty g\nabla_H \cdot \rho\,[\mathbb{W}_o + U(z)\mathbf{i}]\,dz$$

$$= (\partial p/\partial t)_o - \mathbf{i} \cdot \nabla_H \int_o^\infty U(z)\rho g\,dz$$

$$(\partial p/\partial t)(x, y, o, t) = (\partial p/\partial t)_o - \overline{U(z)}\,\mathbf{i} \cdot \nabla_H\, p(x, y, o, t) \;.\;.\;.\;.\;.\;.\;.\; (1)$$

where $\overline{U(z)}$ is the mean averaged with respect to pressure of $U(z)$. Eq. (1) may be interpreted as showing the surface pressure tendency field of the model of an extratropical storm as being made up of two component fields $(\partial p/\partial t)_o$, which may be called the dynamic contribution, yields rises to the east and falls to the west and so tends toward a westward propagation of the low.

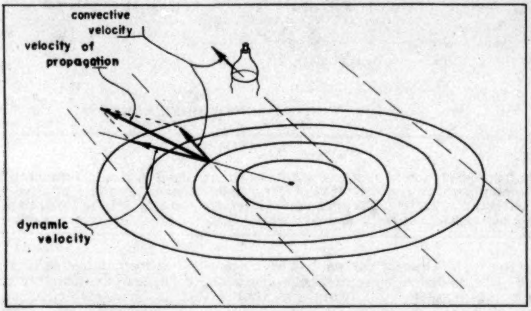

Fig. 1--The convective and dynamic parts of the velocity of propagation illustrated for a section of a water ripple

$- \overline{U(z)}\,\overset{\bullet}{\square} \cdot \nabla_H p\,(s, y, o, t)$ which may be called the convective contribution, effectively advects the surface pressure pattern to the east at the velocity $\overline{U(z)}\,\overset{\bullet}{\square}$.

A rational method for estimating the convective velocity of propagation of a tropical storm would be to fit the components of the wind field near the storm center to polynomials in the position coordinates. The constant terms would then represent the sought for convective velocity. However, in general, too few wind observations have been available near the storm centers. Accordingly running through the literature is found the steering level concept. In the eyes of some meteorologists this is related to an attempt to estimate the motion of the wind stream in which the storm is embedded from winds immediately above or around it but yet sufficiently removed to be independent of the effects of the storm circulation. It is felt that observations so far up or downwind or to the side as to be out of the influence of the storm are also so far away that the basic stream at their positions is no longer representative of that at the storm center. In consequence only the winds above the storm are considered. This leads to the somewhat hazy concept of a steering level as that level where the storm effects are no longer felt but where the wind is still representative of the basic stream. It is of course not at all obvious that such a level will exist. In the eyes of other meteorologists, the winds at this level are of interest because they yield not simply an estimate of the convective velocity alone but rather of the total velocity of propagation. Whichever view is taken, it will be of interest to find to what extent the convective velocity determines the velocity of propagation of actual storms. In this paper somewhat less than this will be attempted. The proposition that the convective component of the velocity of storm propagation is the dominant one and will be tested; on the basis of the limited work reported here, it will be concluded that it is at least probable that this hypothesis must be rejected.

The test--In order to test the hypothesis that the convective part of the velocity of propagation is so important as to virtually determine the direction of propagation, the truth of the proposition will be assumed and a consequent model set up. This will then be tested against all available aircraft observations from Pacific hurricanes of 1947.

Before discussing the model, the concept of the convective velocity will be considered a bit more closely. It will be noted from the examples of the preceding section that the convective velocity is the mean, weighted with respect to pressure, of the velocity of the basic current. Accordingly the winds at lower levels are quite important in its determination. One thus must be cautious about estimating the convective velocity from a wind at a level above a storm. A saving factor for the tropics is that there is comparatively little wind shear with height. It is this which makes reasonable an attempt to estimate the convective velocity from data at but a single level. Partly because the author questions whether it is possible to be both above a hurricane and yet not so high that the shear of the basic current has not been appreciable and partly because of considerations of the availability of data, it is found preferable not to estimate the convective velocity from the wind at a steering level. Instead the two dimensional wind field near the 1500-ft level will be used. This is the flight level used by the reconnaissance aircraft whose data are available for this study. The contribution to the convective component of storm propagation given by a

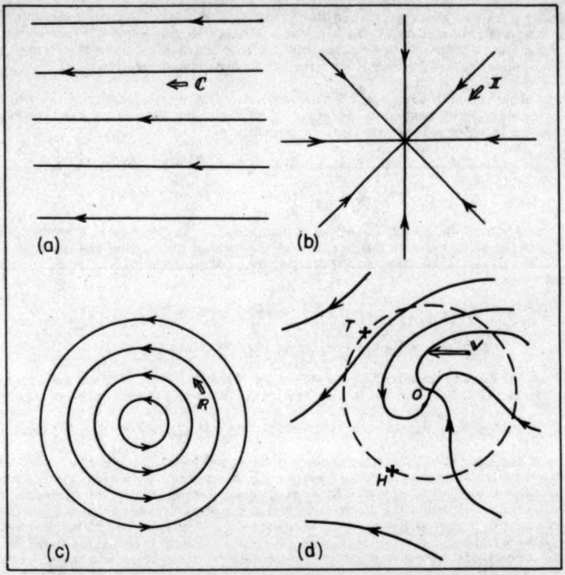

Fig. 2--The model; we assume field (d), the sum of the fields (a), (b), and (c), to approximate that of a hurricane; (It is of interest to note that with our model, the alignment of the field (a) coincides with that of the bisector of the angle HOT between the ray from the storm center 0 to the hyperbolic point and the ray to the point of tangency of the streamlines and the circle about 0 through H.)

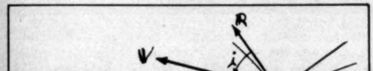

Fig. 3--Notational conventions used for an observation (V, i; r, θ); V is the wind speed, i is the angle of indraft relative to a circle of radius r about the storm center; θ is the azimuth of the point of observation; R, I, and C are the magnitudes, respectively, of the component fields of rotation, indraft and translation

layer centered at this level will be determined and then it will be assumed that because of the small shear in the lower atmosphere and the large weighting given the low layers that this contribution, if given a weight of one, will be a good estimate of the total.

The model--If a constant field of translation of velocity C, a radially symmetric field of rotation of velocity R, and a radially symmetric field of convergence of velocity II, are added vectorially, a field is obtained which may be taken as a first approximation to that of a hurricane (see Fig. 2). It has a dangerous semicircle to the right of the direction of the field of translation. It also has greater angles of indraft in the rear quadrants than in the others. These properties conform rather well to those of an actual hurricane. However, this model is definitely but a first approximation. It fails, for example, to show the spiralling lines of convergence found as squall lines on the radar screen [WEXLER, 1947]; further, it clearly has a symmetric field of convergence and (in the absence of marked density gradients) should show no dynamic speed of propagation. If the hypothesis being tested (that the convective part of the speed of propagation dominates) is a valid hypothesis, then the orientation of the field of translation C of our model should correlate highly with the actual direction of propagation.

Given an observation of wind speed V and angle of indraft i at a point (r, θ) (r and θ are polar coordinates measured with respect to an origin at the storm center and a reference ray pointing to the east), the following equations are obtained (see Fig. 3)

$$V \sin i = I(r) + C \cos (\theta - \alpha) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$V \cos i = R(r) + C \sin (\theta - \alpha) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where I(r) and R(r) are the speeds of indraft and rotation at the distance r, C is the translation speed, and α is the angle between the direction of the field of translation and the east. Thus, given two observations at the same radial distance from the center (V_1, i_1, r, θ_1) and (V_2, i_2, r, θ_2) we have

$$V_1 \sin i_1 - V_2 \sin i_2 = C [\cos (\theta_1 - \alpha) - \cos (\theta_2 - \alpha)] \quad \ldots\ldots\ldots\ldots (4)$$

$$V_1 \cos i_1 - V_2 \cos i_2 = C [\sin (\theta_1 - \alpha) - \sin (\theta_2 - \alpha)] \quad \ldots\ldots\ldots\ldots (5)$$

If we use the well known trigonometric identities for the differences of sines and cosines on the right members of (4) and (5) and then divide one by the other and solve for α, we obtain

$$\alpha = (\theta_1 + \theta_2)/2 + \tan^{-1}[(V_1 \sin i_1 - V_2 \sin i_2)/(V_1 \cos i_1 - V_2 \cos i_2)] \quad \ldots\ldots (6)$$

Results of the test--The directions α were determined (to within 180°) by means of (6) for all possible cases from the report of the 1947 Pacific hurricanes [NAVAER, 1948]. There were 84 pairs of usable observations grouped in 33 rings about the various storm centers. To be usable an observation had to be within the storm field and at a radial distance from the storm center approximately equal to that of at least one other observation (that is, it had to be on the same ring as another). Twice at a given time there were four rings of data, three times there were three rings, seven times there were two rings and the remaining times there was but one ring of usable data.

The 33 rings of data were classified as being in the inner, middle or outer third of the storm circulation. This is an admittedly subjective process. It was done by taking the concensus of independent classifications of the rings by six meteorologists as inner, inner plus, middle minus, middle, middle plus, outer minus or outer.

The orientations of the translatory field (the angles α) and the observed instantaneous direction of the velocity of storm propagation, taken from the published tracks, were correlated. The results are shown in Table 1. Correlations were computed for various groupings: (1) all 84 pairs of usable observations (each leading to an angle α) weighted equally; (2) all rings of data weighted equally; (3) the best ring for each time weighted equally (the best ring was chosen by the rule of priority: middle, outer, inner); (4) middle rings only considered and the determination for each such ring weighted equally; (5) inner rings only; and (6) outer rings only.

Table 1--Correlations between computed orientations of the translatory wind components and the observed direction of propagation of hurricane centers

Item	Unfiltered	Filtered
All 84 pairs of observations	0.74	. . .
All 33 rings	0.50	0.50
Best ring only	0.72	0.71
Middle only	0.75	0.71
Outer only	0.26	. . .
Inner only	0.25	. . .

Certain entries in the table refer to filtered data. These are correlations computed from those rings having four or more pairs of observations. Prior to the computation of these correlations these observations were checked for possible grossly faulty observations as follows. Each observation was deleted in turn and the mean angle α based on the remaining pairs of observations was computed. It was assumed that should a single bad observation exist, more consistent results would be obtained in the computation of α for the particular case when the bad observation was the deleted one. Accordingly if the variation of any subset was less than half of any other, it was assumed that the deletion leading to the subset with small variation from its mean was that of a faulty observation. In actual fact, it turned out that no observations were rejected. Thus the filtered ones are merely those rings with four or more pairs of observations.

Finally it should be noted that all observations have been adjusted by means of the observed motion of the storm for their lack of simultaneity.

Conclusions--The results of the test yielded no correlations with squares larger than 0.56. Since the square of the correlation may be interpreted as the fraction of the variation of one variable predictable from the other [HOEL, 1947], this means that not more than 56 per cent of the variation in the observed directions of propagation, which ranged from 116° to 200°, is predictable from the computed angles α. This can be attributed to but two general causes; failure of the model to fit reality and observational errors. In order to see the uncertainty introduced by observational errors one could perform the experiment of entering the formula with synthetic data which is the sum of observations computed to fit the model exactly plus randomly selected errors of the order of magnitude believed to exist in practice. However, at this stage with so little reconnaissance data available, it is outside the scope of this paper to apportion the relative variability due to observational errors on one hand (perhaps quite large) and to failure of the model on the other. The model is one which should give a good approximation in the event of the validity of the hypothesis that the convective portion of the velocity of propagation is the dominant one. Clearly one can draw the tentative conclusion, on the basis of the limited data tested, that the direction of propagation cannot be predicted from the convective velocity alone. As a matter of personal opinion, it is the author's belief that not all of the remaining variability will be found to be due to observational errors but, rather, that the dynamic contribution will be found to be significant.

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SUNRISE TEMPERATURE DIPS

Ronald L. Ives

Abstract--Temperature depressions of as much as five degrees F, in the Great Basin and Rocky Mountain regions, during the hour centered about sunrise, appear to be closely related to local air drainages. These sunrise temperature dips may produce errors in mean temperatures as now computed; and lead to possible misadjustment of thermographs.

Introduction--Wartime field studies at Dugway, Utah, disclosed that air temperatures commonly dipped sharply several degrees during the hour centered about sunrise. These sunrise temperature dips coincided temporally with the morning turnaround of the local surface winds, and were commonly, but not invariably, accompanied by a barometric jitter. Onset of the sharp temperature decline commonly occurred when the east sides of nearby mountains were first illuminated by the Sun, and upward inflection of the temperature curve occurred when the pyrheliometer began to register.

Because Dugway is an atypical area both geographically and culturally, comparison of instrumental records from that station with those from standard and long-established Weather Bureau stations in the Rocky Mountain and Great Basin regions was deemed advisable, and disclosed similar sunrise temperature dips at other locations. Samples of these dips, as recorded at Salt Lake City, Utah, are shown in Figure 1. Similar dips were noted at other stations near mountains, particularly at Denver and Boulder, Colorado.

Magnitudes of dips--Depths of one-hour temperature dips occurring about sunrise at Dugway, Salt Lake City, and Denver seldom exceed five degrees F, when recorded by a standard Weather Bureau thermograph which responds to temperature changes rather slowly.

In a number of instances, the minimum temperature, as indicated by the minimum thermometer, was one or two degrees below that shown by the thermograph, even though both instruments agreed under "steady state" conditions.

Parallel measurements with a fine-wire thermocouple and an electronic recorder [GRINNELL, 1945; IVES, 1946] showed that the depths of the temperature dips, as indicated by a massless and instantaneous instrument, were sometimes double those shown by a standard thermograph; and that, regardless of the numerical value of the dip, the difference between the indicated thermocouple and thermometer minimums was almost exactly 3/2 of the difference between indicated thermometer and thermograph minimums. This suggests that the discrepancies in indicated minimums are due in large part to the varying response rates of the instruments.

Mean values of the sunrise temperature dips, as determined from thermograph records for three consecutive years from Dugway, Salt Lake City, and Denver, are shown in Figure 2. The Dugway record is from a thermograph located in a weather shelter outside of the cantonment area, and surrounded by almost barren clay flats, about 1500 ft from the nearest structure. Other records are those supplied on microfilm by the U. S. Weather Bureau. Shape of the curves suggests that longer record runs are desirable, when data permit.

A number of tests show that the sunrise temperature dip is greatest, on any given day, at low levels, such as one to three inches above the ground, where it may be double that at five feet, the customary elevation of instruments in shelters; and that it becomes very much less at higher levels, such as 50 ft above the ground.

Relation of dips to other meteorological phenomena--Day-by-day comparison of dip magnitude with other weather phenomena recorded at the same station discloses a number of relations, not all of which are necessarily significant Sunrise temperature dips are best developed after clear nights with little wind, and are not usually present after cloudy nights, or when there is morning fog.

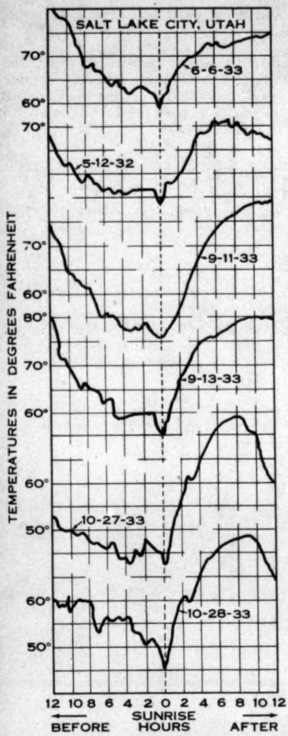

Fig. 1--Tracings of thermograph curves from Salt Lake City, Utah, showing sunrise temperature dips; data from U. S. Weather Bureau

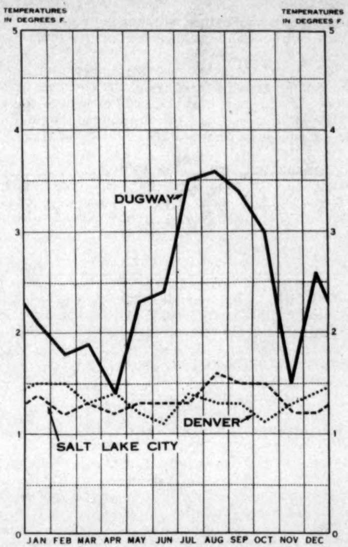

Fig. 2--Mean magnitudes of sunrise temperature dips, for three stations, based on three consecutive years of thermograph traces from each station

These dips appear to be related to valley air drainage, the upward inflection of the temperature curve coinciding with the reversal of the valley winds from downvalley (night condition) to upvalley (day condition). This coincidence is even more marked when fast response instruments are used.

In a general way, but not with 100 per cent correlation, depth of the sunrise temperature dip varies directly as the magnitude of the valley air drainage, both at a single station, and from one station to another. An outside wind sufficient to suppress the local valley air drainage is usually also accompanied by a suppression of the sunrise temperature dip.

Although sunrise temperature dips are usually accompanied by a barometric jitter, or oscillation, sometimes amounting to 0.15" of mercury (5 mbs), instrumental tests do not disclose whether this jitter is caused by temperature changes, or by wind changes, or by a combination of factors. Rough computations suggest that the magnitude of the pressure change is too great for it to be caused by change of temperature alone.

Significance of dips--Sunrise temperature dips, as noted in the Great Basin and Rocky Mountain regions, appear useful as indicators of the presence of valley air drainages, and may, after further study, have additional use in evaluating the magnitudes of these air drainages. Where sunrise temperature dips are present, the mean temperature, as computed by conventional methods (one-half the sum of the maximum and minimum temperatures) may be in error by about half the magnitude of the dip. Because thermograph records from most areas are subject to vagaries other than sunrise dips, which may or may not be compensating causes of error, a blanket arbitrary correction of the mean temperature appears inadvisable.

In areas subject to sunrise temperature dips, particularly when their duration is short, and their depth great, correction of the thermograph by comparison with the minimum thermometer at about sunrise may lead to misadjustment of the recorder, due to inequalities in the response rates of the instruments, and the checking of thermal instruments should be postponed until such a time as the change of temperature is slow and steady, such as in mid-forenoon.

Acknowledgments--The writer is indebted to G. K. Greening of the U. S. Weather Bureau, Salt Lake City, Utah, for helpful discussions of sunrise temperature dips and other thermal vagaries, and for access to the voluminous records of that office; and to S. W. Grinnell, now of Stanford University, formerly of Section 10, NDRC, for many instrumental records and check observations.

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NIGHT SKY BRIGHTNESS AT LATITUDE 52°

B. W. Currie

Abstract--Observations of the night sky brightness at Saskatoon, Canada, latitude 52° 11′ N, indicate that the brightness on clear nights without visible polar aurora is appreciably greater than at lower latitudes. Large differences between the average brightnesses for nights without apparent differences in sky and atmospheric conditions, a seasonal variation in the average brightness corresponding approximately to the seasonal occurrence of polar aurora, and observations of sky luminosities when polar aurora is present suggest that this increase in brightness is due to light of polar auroral origin.

Introduction--Numerous measurements of the brightness of the night sky have been made at Saskatoon, Canada, (52° 11′ N, and 106° 41′ W), at periodic intervals during the past 14 months. These were made by the writer as his contribution to a project initiated by E. O. Hulburt, Naval Research Laboratory, Washington, D. C., to obtain values of the night sky luminosity from higher latitudes which are comparable with each other and with earlier values from lower latitudes. The photometer (similar in type to that used by HULBURT [1948, 1949; RICHARDSON and HULBURT, 1949] at lower latitudes and by others participating in this project), was supplied by the Naval Research Laboratory, In addition to a calibration for converting the settings on the photometer into millimicrolamberts (mμL), a brightness standard for checking the calibration was supplied.

Since no observations of night sky luminosities had been made previously in this part of Canada, series of measurements were made under a variety of ground, atmospheric, and sky conditions. These included bare and snow-covered ground, scattered and continuous cloud, light fogs consisting of both ice and water particles, twilight, moonlight, and displays of polar aurora, as well as under conditions which appeared to approximate closely the ideal one for determining the luminosity of the night sky. In addition to providing a certain amount of basic information, some of these observations provided criteria for selecting occasions when the observations were least affected by attenuation of the light by particles in the lower atmosphere. This was particularly necessary since it soon became apparent that the night sky light in this region is subject to large variations.

Individual observations in a short series on a particular part of the sky seldom vary by more than 15 per cent from the average for the series. On the other hand, observations on different nights with apparently the same sky and atmospheric conditions often differ consistently for all parts of the sky by as much as 60 per cent. This may be due to the proximity of Saskatoon to the zone of maximum auroral occurrence. Although Saskatoon is about 550 mi south of the accepted position of the isochasm of maximum frequency, polar aurora occurs at least once in every three nights, on an average; for example, during 1949 an auroral recorder indicated zenithal aurora on 148 out of 222 nights with either little or no cloud.

Night sky brightness--Table 1 gives values of the night sky brightness in mμL, corresponding to a range of altitudes from 5° to 90°. Column (a) gives the average value for six nights in late July, August, and early September; (b) for five nights in late September, October, and November; and (c) for four nights in December and January. These are for nights which, in the writer's opinion, came closest to the ideal night: clear skies, a minimum of attenuating particles and no obvious luminosity due to polar aurora. With few exceptions, the weather on these nights was characterized by the presence of polar continental air sweeping southward at the rear of a cyclonic disturbance and producing excellent visibility. The observational procedure consisted of making sweeps from 5° above the horizon to the zenith in directions corresponding to the first eight points of the compass except N and NE. These were excluded because they are the directions in which polar aurora is first observed at Saskatoon. Observations were made at 5° intervals for the lower altitudes and 15° intervals for the higher; and six settings of the photometer were made for each position. In analyzing the observations for a night, the average luminosities for each direction were plotted against the corresponding altitudes. A comparison of these plots made it possible to eliminate high values of the luminosity in particular plots due to the presence of the Milky Way and bright stars in the field of the photometer. A composite graph for the night was then drawn by averaging the values for all directions as read from the individual plots. These graphs were used in turn to get the average values given in Table 1. Approximately 250 observations of the sky brightness are involved in the observations for each night, or approximately 1500 for column (a), 1250 for column (b), and 1000 for column (c).

Table 1--Night sky brightness at Saskatoon, Canada

Altitude	(a)[a]	(b)[a]	(c)[a]	(d)[b]	(e)[b]	(f)[c]	(g)[c]
°				SW	NE	SW	NE
5	206	338	294	243	859	. . .	664
10	262	408	353	336	2970	1090	2780
15	287	414	372	368	. . .	1180	. . .
20	276	385	343	334	459	980	1585
30	235	334	298	308	318	785	1250
40	209	294	269	277	261	785	1220
50	198	271	249	234	224	930	1195
60	194	265	241	215	206	1230	1215
70	189	260	230	211	196	1210	1190
80	186	255	226	206	193	1225	1180
90	185	254	221	198	198	1210	1210

[a] Average brightnesses in mμL, for late summer, autumn and winter, respectively, for altitudes from 5° to 90° above the horizon.

[b] Typical brightnesses with a quiet auroral arc 10° above the north-east horizon during August.

[c] Corresponding brightnesses during apparently similar displays later in the year.

The average brightnesses for each of the three periods are all higher than the ones reported for lower latitudes. In addition, they indicate a possible annual variation of the night sky brightness, minima occurring during summer and winter, and maxima during autumn and spring. Since this variation is the same as that for the occurrence of polar aurora to the south of the auroral zone, it adds weight to the suggestion that the normal night sky light, typical of lower latitudes, is augmented at higher latitudes by a luminosity of polar auroral origin.

Luminosity with polar aurora--Columns (d) and (e) of Table 1 give the sky luminosities for a typical sweep of the photometer from the southwest horizon through the zenith to the northeast horizon when an auroral arc was visible at about 10° above the northeast horizon. Columns (f) and (g) are for an almost similar situation, but with the auroral arc accompanied by an added auroral luminosity which extended far to the south. The first pair is for the night of August 26-27, and the second for October 23-24. During August all quiet auroral displays to the north were characterized by luminosities of the first type. During September and October the luminosities for similar auroral displays were almost exclusively of the second type. An almost complete lack of auroral displays since late November has made it impossible to determine whether or not this change in luminosity is a seasonal one.

During a very active and bright display on the night of August 3-4, individual features in the display attained a brightness of 1.9×10^{-5} lambert, and the illumination on a horizontal surface 1.7×10^{-3} ft candles. While these are the highest values which have been observed with the photometer, previous experiences of the writer indicate that much higher values occasionally occur. Intense rays approximate the brightness of the full moon. During widespread, active displays pencilled notes will become momentarily visible without artificial illumination.

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A SURVEY OF NIGHT SKY BRIGHTNESS AT COLLEGE, ALASKA

John B. Wilcox

Abstract--The results of a series of night sky brightness determinations, made by means of the NRL Photometer Model 2, No. 3, are given. Data are compiled as mean values of brigthness in mμL for specific points in the sky over a period of several weeks. Observations were made on clear, moonless nights in both the presence and absence of visible aurorae. A brightness contour map of the compiled data is presented.

The present survey was undertaken primarily to determine how light from the arctic night sky compares in intensity with values reported from lower latitudes. For the sake of brevity we have defined the following quantities after HULBURT [1948]: B, brightness of a point in the sky in mμL as viewed with the eye adapted to the illumination level under observation; and P, altitude in degrees above the horizon of a point in the sky. A low brightness photometer furnished by the Naval Research Laboratories was used for all brightness determinations.

Readings were taken from a reasonably clear area adjacent to the Ballaine's Lake Field Station of the Geophysical Institute. Lights of the city of Fairbanks were visible in the distance, as were the lights of two radio towers at distances of approximately one and two miles. That conditions were not ideal is obvious; however, concession was necessarily made to the availability of heated shelter, for the temperature ranged to -50° F during the course of the survey. The elevation of the point of observation was about 500 ft above sea level and its location was 64° 52′ N and 147° 49′ W.

Observations were made during the period from January 16 to March 11, 1950. Brightness determinations were made at points from P = 15° to the zenith and at azimuths of geographic north and intervals of 45° therefrom (eight points of the compass). In each case, observations were made during the early part of clear, moonless nights well after the end of twilight. In the majority of instances a faint, quiescent auroral arc was evident in the low northeast. The maximum P for which this arc was observed throughout the course of the survey is indicated in the compiled data. In no case was the auroral display of great magnitude or of visibly changing form or position.

Telepone communication was maintained between the observer and an assistant who recorded the data, to avoid interfering with the dark adaptation of the observer's eyes, which was necessary for the matching of low brightnesses. A red filtered flashlight provided light essential to the observer's reading of the instrument.

Data were divided into two groups, those taken when aurora was evident and those taken in the absence of any visible aurora. Tables 1 and 2 give the mean brightnesses of the previously mentioned points for each of these groups respectively. Contour maps constructed from the data of these tables are shown in Figures 1 and 2 respectively. Observations in the vicinity of the Milky Way were discounted in compiling the data. Zodiacal light was not observed.

It is believed that the patchy appearance of Figure 2 is a statsitical variation, for clear, moonless nights in which no aurora could be seen were rare and data for Figure 2 were, consequently, few. It seems reasonable to assume that sufficient data of the type taken would result in a more uniform series of contours.

It appears that values of B recorded here for nights without aurora are in good agreement with those reported [RICHARDSON and HULBURT, 1949] at Bocaiuva, Brazil (17° 13′ S, 43° 41′ W) during April and May of 1946. One possible significant difference is evident, however; the decrease from higher values near the horizon to lower values in the zenith is more gradual in the case of the College data. This could indicate a difference in the height of the emitting layer (or layers) at the two stations. VASSY and VASSY [1948] analyzed similar data taken at Pic du Midi Observatory (42° 56′ W, 0° 40′ W), taken for spectral components, and found that no single emitting layer could account for their experimental curves. It appears, according to their method of analysis, that either one or more of the emitting layers must be lower at College than at Bocaiuva or that the intensity of one or more layers must differ.

Table 1--Night sky brightness at College, Alaska, Aurora present

Azimuth	P	Mean B
		mμL
N	15	427
	30[a]	247
	45	186
	60	167
	75	151
S	15	270
	30	253
	45	226
	60	194
	75	164
E	15	273
	30[a]	191
	45	158
	60	143
	75	138
W	15	250
	30	216
	45	197
	60	. . .
	75	155
NE	15	369
	30[a]	250
	45	177
	60	152
	75	145
SW	15	215
	30	222
	45	225
	60	202
	75	165
NW	15	318
	30[a]	263
	45	182
	60	183
	75	147
SE	15	264
	30	214
	45	182
	60	155
	75	114

---

[a] Maximum P of visible aurora for the given azimuth.

Table 2--Night sky brightness at College, Alaska, Aurora not present

Azimuth	P	Mean B
		mμL
N	15	246
	30	216
	45	182
	60	160
	75	152
S	15	276
	30	253
	45	196
	60	214
	75	188
E	15	247
	30	217
	45	174
	60	154
	75	148
W	15	249
	30	225
	45	199
	60	224
	75	143
NE	15	220
	30	145
	45	130
	60	130
	75	122
SW	15	210
	30	213
	45	196
	60	179
	75	170
NW	15	259
	30	236
	45	188
	60	154
	75	133
SE	15	258
	30	188
	45	152
	60	132
	75	133

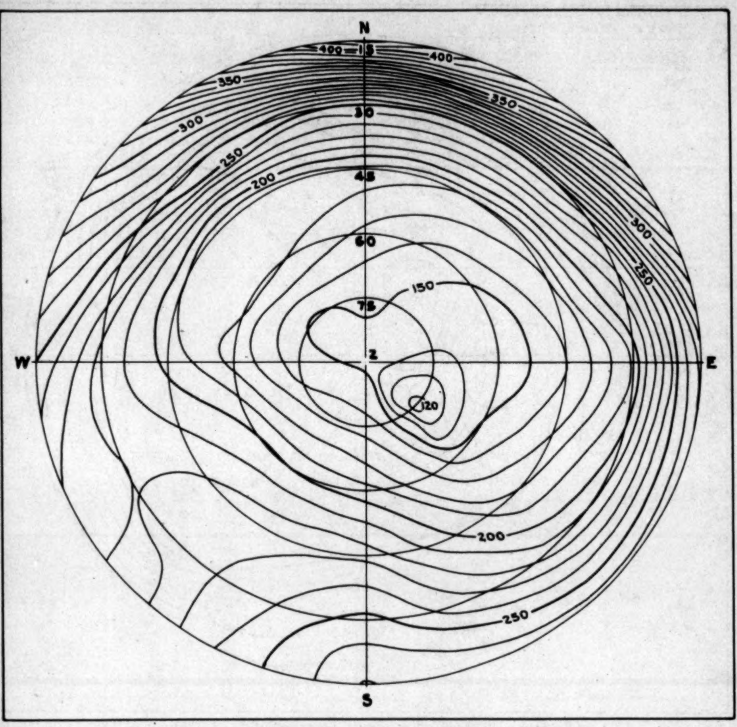

Fig. 1--Contours of equal brightness of night sky light at College, Alaska, in millimicrolamberts, aurora present

Values of B at College are greater than those reported from Whiteface Mountain Observatory (44° 30′ N, 73° 58′ W), Bikini (11° 35′ N, 165° 30′ E), and Maryland (38° 32′ N, 76° 10′ W), as compiled by HULBURT [1948]. This is due probably to better visibility at College.

It was noticed during the course of the study that the darkest area of the sky seemed consistently to be slightly east of the zenith. That this was actually the case is borne out by the contour maps. Since care was taken that no residual twilight was present during the observations, no explanation of this phenomenon is evident.

Aside from the rather patchy nature of Figure 2, the two maps agree very well in the directions for which no aurora was observed. This confirms the fact that auroral displays were localized and suggests that brightness values well removed from the zone of auroral activity could be compiled along with those taken on nights without aurora.

Acknowledgments--The author would express his gratitude to E. O. Hulburt of the Naval Research Laboratories for his kind loan of the photometer used in this survey, and to J. G. Cazort for his assistance in gathering data.

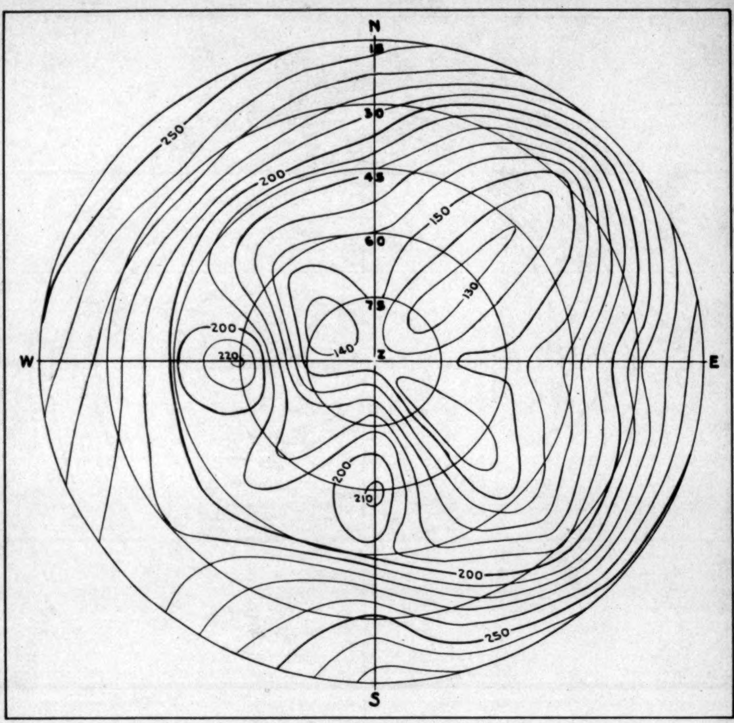

Fig. 2--Contours of equal brightness of night sky light at College, Alaska, in millimicrolamberts, no aurora present

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NIGHT SKY BRIGHTNESS IN MARYLAND AND GREENLAND DURING 1949

C. A. Pearson and M. J. Koomen

Abstract--The measurements relate chiefly to the light ascribed to the upper atmosphere, as a function of altitude above the horizon. They do not show any important changes with latitude. Night-to-night and long-term fluctuations are revealed by the studies, which are still in progress. Possible correlations with other cosmic data have been examined; the only correlations thus far developed are with the long-term trends of solar coronal indices as observed at Climax, Colorado.

Visual measurements of night sky brightness were made from stations located at various latitudes north of the equator. The photometer used for the measurements employed a radium-activated luminous phosphor button, and is described in another paper [TOUSEY, KOOMEN, and DUNKELMAN, 1950]. Measurements were made only on clear cloudless nights with no moon, twilight, or noticeable aurora; the Milky Way was avoided in the measurements. Therefore, the measured brightnesses refer mainly to the visible radiations from the permanent aurora originating in the upper atmosphere, other radiations such as light from faint unresolved stars and from interplanetary space being a small part of the total [HULBURT, 1949].

Brightness readings were taken from horizon to zenith at various altitude angles, H, for several azimuths. The measured values of night sky brightness, B, were plotted against H.

From measurements on 11 nights on Chesapeake Bay, Maryland (latitude 38° N), it was found that the sky brightness, averaged between December 1, 1948 and April 1, 1949, was about 200 millimicrolamberts (mμL) at 20° above the horizon and decreased gradually to about 175 mμL at the zenith.

Measurements on five nights at Sondre Stromfjord, Greenland (latitude 67° N), during February and March 1949 made by the Air Weather Service of the USAF, gave sky brightness values not differing greatly from those obtained at Chesapeake Bay during this period. The average brightness was 225 mμL at 20° above the horizon, decreasing to 140 mμL at the zenith.

The above brightness values are similar to those obtained a number of years previously for latitudes from 17° S to 45° N [HULBURT, 1949]. Thus we have not found any important variations of night sky brightness with latitude. In the 1949 observations, night-to-night fluctuations of sky brightness were observed at both the Maryland and Greenland stations. It is not known whether these were the result of changes in atmospheric clarity at the stations or of actual fluctuations in the light from the night sky. In addition to the night-to-night fluctuations, the data showed a long-term trend in night sky brightness. The Maryland zenith sky brightness averaged 190 mμL during December 1948 and January 1949 and then fell gradually to 130 mμL at the end of March. The Greenland data, although taken only during February and March, showed the same trend. Again it is not known whether this trend was due to a seasonal fluctuation in atmospheric clarity or to actual seasonal fluctuation in emission from the night sky.

An attempt was made to correlate the observed night sky fluctuations with some of the evidences of solar fluctuations during the same period of time. There appeared to be no connection between night sky brightness fluctuations and other phenomena such as fluctuations in ionospheric character, geomagnetic K-figures or sunspot numbers. It was found, however, that the seasonal fluctuation of the intensity of the solar corona, as observed at Climax, Colorado, had somewhat the same trend as the observed sky brightness fluctuations. This correspondence between coronal and sky brightness fluctuations appeared only in the long-term trend. There was no apparent connection between day-to-day coronal fluctuations and night-to-night sky fluctuations.

Night sky brightness measurements are continuing in 1950. The results thus far are fragmentary but are similar to those of 1949. The measured sky brightnesses are similar, and suggest the same trend. They show no variation of night sky brightness with latitude.

At Chesapeake Bay, measurements of the illumination from the Pole Star are also being made in order to determine whether there is any relation between the night sky brightness and the transmission of the atmosphere overhead.

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A VISUAL PHOTOMETER FOR MEASURING THE BRIGHTNESS OF THE NIGHT SKY

R. Tousey, M. Koomen, and L. Dunkelman

Abstract--The instrument described is suitable for night operation in high latitudes. It has no electrical circuits or lamps, and has a lower brightness limit of about .010 μL. A phosphor button serves as the comparison source.

For the survey of the brightness of the night sky at high latitudes simple visual photometers were constructed which would hold their calibration, and would be light and easy to operate at night in cold weather. A photometer with comparison field illuminated by a self-luminous button was ideally adapted for this purpose, since it was not encumbered with lamps, meters, and batteries. A photometer of this type was first described by RAYLEIGH [1924]. Our particular design was patterned after a photometer constructed by Brian O'Brien, P. Taylor, M. Koomen and others at the University of Rochester in 1941 and has been used at the Naval Research Laboratory since 1942. A special modification was designed for use in measuring the night sky.

Figure 1 shows the optical system of the photometer. The observer looks through the circular apertures A_1, A_2, A_3, at the field whose brightness is to be measured. When this is the sky, it is seen by reflection in the 90° prism P. This prism can be rotated about the horizontal line of sight so as to view points in the sky at different altitudes without tipping the photometer. Azimuths are set by rotating the photometer as a whole on its supporting tripod.

The comparison field occupies the center of the field of view and is reflected in by the 45° mirror M, which is an ellipse of eccentricity equal to $1/\sqrt{2}$. The field seen in M is the opal glass O, which is illuminated by the radium activated self-luminous button B; this is a two-inch Navy deck marker button and is carried on the end of an arm. The brightness of the comparison field is varied by rotating the arm and so changing the distance between B and O. The position is read from a dial. Lens L simply enables the observer to focus on the mirror M so that the comparison and test fields are fairly sharply delineated.

The instrument is calibrated against a photometer with comparison field at 2360° K color. A secondary standard source is provided with each instrument for checking purposes and consists of

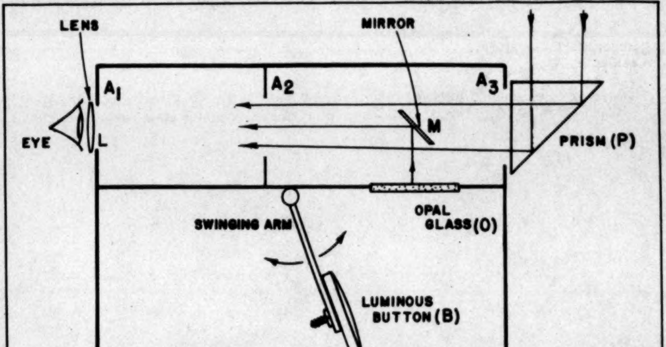

Fig. 1--Optical system of visual photometer an opal glass illuminated by a standard lamp operating at 2360° K which can be mounted on the photometer in place of prism, P. Experience has shown that the self-luminous buttons are usually as stable as the standard lamps and meters. The button brightness, by test in a cold chamber, does not change measurably at low temperatures.

The field of view in the sky observed in the photometer is 11° in diameter and the comparison field occupies the center 6°. The photometer covers a brightness range from 2 μL to about 0.010 μL. By inserting a wire screen over the aperture A_3 the upper limit can be extended to 20 μL.

Figure 2 shows the photometer in operating position. It is leveled with the small circular bubble level on top. The chamois cover over the end is intended to keep the observer's face from touching the metal box in case of very cold weather. To measure illumination on a horizontal surface, a standard piece of white matte paper is placed in the ground, the prism is removed and the photometer is turned so as to view the paper direct.

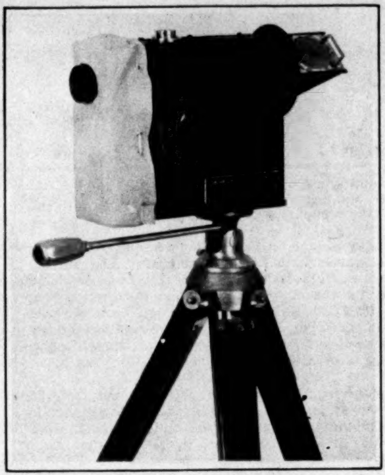

Fig. 2--Visual photometer mounted on tripod

The photometer can be changed into a Maxwellian view telephotometer by inserting a lens at A_3. This lens has a focal length such that it images a distant object on the surface of the eye. At the same time a small aperture (3 mm diameter) is inserted at A_1. The telephotometer when properly calibrated can be used to measure the illumination produced by stars. In particular, it is used to measure the illumination produced by the North Star. From this it is possible to calculate the vertical transmission of the atmosphere, and to determine whether it exerts an influence on the brightness of the night sky.

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REFRACTION OF SHALLOW WATER WAVES: THE COMBINED EFFECT OF CURRENTS AND UNDERWATER TOPOGRAPHY

(Contribution from the Scripps Institution of Oceanography, New Series, No. 470)

Robert S. Arthur

Abstract--A solution is given for determining the refraction effect according to Fermat's Principle for the case of shallow water waves moving in any given distribution of current and depth. Application is made to an analytic model of an intense rip current, and the results are compared to actual rips.

Introduction--UNNA [1942] and SVERDRUP [1944] have considered the case of deep water waves meeting a following or opposing current, and JOHNSON [1947] has discussed the refraction of deep-water waves encountering a uniform current which moves at an angle to the wave direction. In the present paper the combined refraction effect of underwater topography and of a current is discussed for the special case of shallow water waves whose velocity, c, is given to a satisfactory approximation by the expression

$$c^2 = gh \qquad (1)$$

where g is the acceleration of gravity and h is the depth. Horizontal variations in current and depth give rise to a refraction problem involving a moving medium and a relative wave velocity which varies horizontally. The problem is analogous to that of determining the minimal flight path of an airplane flying in a variable wind field. A solution to the minimal flight problem is used to determine the refraction effect of a current distribution of the type represented by rip currents. The method is also applicable to tide rips and longshore currents in shallow water, and the former, particularly, may be expected to show refraction effects of importance [JOHNSON, 1947].

The close correspondence between certain problems of minimal flight and refraction was noted by FRANK [1918]. The determination of the minimal flight path in two dimensions for a fixed air speed was first accomplished by ZERMELO [1931]. His results were subsequently extended, and DE MIRA FERNANDES [1932] has obtained the solution for an air speed which varies in space and time. ARROW [1949] includes a review of the literature in his recent paper.

Solution--Diagrams which show the refraction effect of bottom topography have proven useful in the study of waves and related phenomena in the nearshore zone [MUNK and TRAYLOR, 1947]. It has been customary on such diagrams to draw a network of lines which are perpendicular to the crests and are, therefore, designated as orthogonals. The orthogonals correspond to the rays in optical refraction, and according to Fermat's Principle, they represent paths of least time. It is, then, Fermat's Principle which is the basis for the analogy between the refraction and minimal flight path problems. In our problem the direction of wave propagation and wave speed relative to the current correspond to the heading and air speed, respectively, of the aircraft. The current corresponds to the wind field, and the ray path corresponds to the minimal flight path. Since the rays are not perpendicular to the wave crests in the presence of a current, the term "orthogonal" is no longer appropriate.

The refraction problem is next formulated mathematically. We consider the two-dimensional case and let the rectangular coordinates of a moving point on a ray be given as functions of the time, t, that is $x = x(t)$ and $y = y(t)$. Let the x- and y-components of the horizontal component of the current velocity, v_H, be v_x and v_y respectively, and let θ designate the angle between the relative wave velocity, c, and the positive x-direction. Then $x = x(t)$ and $y = y(t)$ must satisfy

$$\left.\begin{aligned} dx/dt &= v_x + c\cos\theta \\ dy/dt &= v_y + c\sin\theta \end{aligned}\right\} \qquad (2)$$

According to Fermat's Principle, the ray between two points P_1 and P_2 (see Fig. 1) is the path of least travel time, subject to (2) which relate the variables.

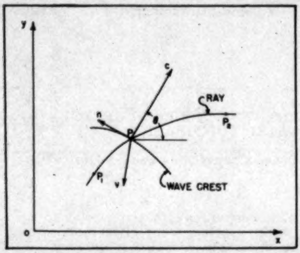

Fig. 1--Schematic diagram showing ray, wave crest, and velocity vectors

This formulation of the problem is identical mathematically with that for the determination of the flight path of least time for the airplane. ZERMELO'S [1931] result gives as the condition to be satisfied for the minimal path

$$d\theta/dt = -(\partial v_x/\partial y)\cos^2\theta + (\partial v_x/\partial x - \partial v_y/\partial y)\cos\theta\sin\theta + (\partial v_y/\partial x)\sin^2\theta \quad . . (3)$$

where the assumption is made that c is constant (uniform depth) and that $c^2 > v_H{}^2$. If v_c denotes the component of v_H in the direction of c, (3) has the form

$$d\theta/dt = -(\partial v_c/\partial y)\cos\theta + (\partial v_c/\partial x)\sin\theta . . (3')$$

The assumption that c is constant is generally not appropriate in wave refraction applications. Zermelo's result is easily generalized for the case where c is a function of x and y and the result is [DE MIRA FERNANDES, 1932]

$$d\theta/dt = -[\partial(v_c + c)/\partial y]\cos\theta + [\partial(v_c + c)/\partial x]\sin\theta \quad \ldots\ldots\ldots\ldots (4)$$

If n denotes the direction of the tangent to the wave crest (see Fig. 1), (4) takes the simple form

$$d\theta/dt = -\,\partial(v_c + c)/\partial n \quad \ldots\ldots\ldots\ldots (4')$$

Condition (4′) can be stated as follows:

> The rate of change of wave direction is numerically equal to the variation or shear along the crest in a scalar field composed of the relative wave speed and the current component in a direction normal to the crest; the direction of wave advance turns toward the direction of negative shear along the crest.

Equations (2) and (4) form a system of three ordinary differential equations with three dependent variables x = x(t), y = y(t), and θ = θ(t). Since the equations are first order, a solution for any desired ray and the crest orientation along the ray can always be obtained, although numerical methods may be necessary. The problem of determining the refraction effect is thereby solved, and advantageously, since the rays are determined directly. A graphical method of solution similar to that given by JOHNSON, O'BRIEN, and ISAACS [1948] for refraction by bottom topography alone can be applied to the equivalent system consisting of (2) and (4′).

Application--The method of solution as outlined is applied to obtain the wave refraction effect of a current distribution of a type represented by rip currents, which are narrow, fast flowing currents directed away from shore. The method permits the use of an actual rip current distribution, but no detailed observations of such a distribution are available. However, observations of the flow pattern and the velocities in the "feeders" and "neck" are available [SHEPARD, EMERY, and LA FOND, 1941; SHEPARD and INMAN, 1950], and these have been used as a guide in choosing an analytic representation.

The bottom is assumed to have a uniform slope of 1:50, and the coordinate axes are as shown (see Fig. 2). The assumed x- and y-components of the current are

$$\left.\begin{aligned} v_x &= -3.60[2 - (y/250)^2]\cdot F(y/250)\cdot\int_0^{x/25} F(\alpha)d\alpha \\ v_y &= 0.1442y \cdot F(y/250)\cdot F(x/25) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (5)$$

where the length and time units are feet and seconds, respectively, and where the function F is defined by

$$F(\alpha) = (1/\sqrt{2\pi})\, e^{-\alpha^2/2}$$

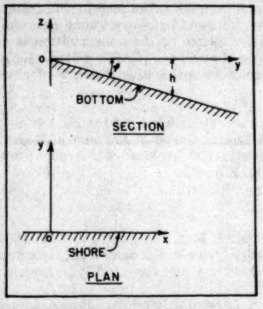

Fig. 2--Coordinate axes for example

Equations (5) have been chosen to satisfy a continuity relationship and to give a current distribution of a type represented by rip currents (current vectors, Fig. 3, are derived from (5)). The maximum value of the current is 3.5 ft/sec and width of the neck in which the outward flow is appreciable is 100 ft (see Fig. 3). If the equation of continuity $(\partial v_x/\partial x) + (\partial v_y/\partial y) + (\partial v_z/\partial z) = 0$ is integrated from surface to bottom and the boundary condition that the normal velocity be zero at surface and bottom is applied, the resulting form is

$$\partial v_x/\partial x + \partial v_y/\partial y = - v_y/y \quad \ldots\ldots \quad (6)$$

where it has been assumed that v_x and v_y are uniform with depth. Equations (5) satisfy (6).

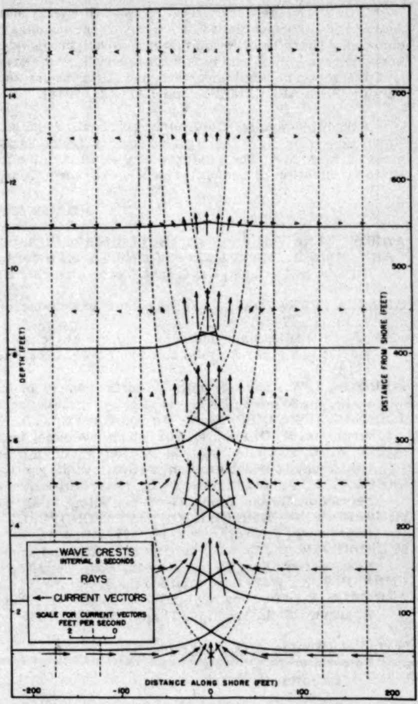

Fig. 3--Diagram showing computed results of refraction by both currents and underwater topography

Several rays (dashed lines, Fig. 3) have been constructed by solving (2) and (4) numerically, with the initial condition that the rays be normal to shore at a depth of 15 ft. Values of $\theta(t)$ have been used in constructing segments of the wave crests along the rays at intervals of eight seconds. The crests (solid lines, Fig. 3) have been completed by interpolation. The solid lines shown can be interpreted to be either successive positions of the same crest at intervals of eight seconds or since the wave period is constant, the position of the wave crests at a fixed time in a train of waves of period eight seconds. The free ends of the crests are in the proximity of caustics, and the solution has not been continued through these regions because refraction principles alone are inadequate for determination of the wave features under these circumstances. Ordinarily the width of the current at the neck is not large compared to a wave length and the additional complication of wave diffraction may also modify the solution.

Waves in actual rip currents in some cases show features which are similar to those in the illustrated refraction pattern. The crests generally show a lag in the neck of the current and the crisscross pattern of crests may sometimes be seen in very shallow water. The current velocities and velocity gradients chosen for the model are typical only of the strongest rip currents. Many rips seem to cause only a gentle retardation of the wave crests with no apparent crisscross pattern; this difference may be the result of much lower current velocities and velocity gradients. Another factor is the channel which results from erosion of a sand bottom below the currents in the feeders and neck. The effect on wave velocity of the greater depth in the channel tends to offset the effect of current shear in refracting the waves.

Quantitative deductions regarding wave energy and height are not attempted in the example. As the rays proceed toward shore, divergence occurs first, followed by convergence near the caustics. There is evidence that simple height considerations based on the conservation of energy between rays do not hold for breaking waves in the presence of currents [CARR, 1949]. Reflection and diffraction become important, and observations show lower waves inside the neck of rips than outside [SHEPARD, EMERY, and LAFOND, 1941].

Acknowledgments--This work represents results of research carried out for the Office of Naval Research, Navy Department, and the Beach Erosion Board, Corps of Engineers, Department of the Army. The existence of a solution to the minimal flight problem was brought to the writer's attention by one of Arrow's co-workers, George E. Forsythe.

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AN EXAMPLE OF THERMAL CONVECTION

Henry Stommel

(Contribution from the Woods Hole Oceanographic Institution, No. 524)

Abstract--The first few terms of a series development of the Sandström model of thermal circulation are presented. The solution is convergent only under special conditions.

In lieu of a theoretical description of the velocity field induced in a basin of water by temperature differences maintained at the surface, a qualitative picture due to Sandström is often quoted in texts on oceanography and meteorology [for example, SVERDRUP, 1942]. Although the Sandström model probably is inapplicable to the thermal circulation in the ocean, it is of interest itself because it is governed by essentially non-linear equations. The purpose of this note is to present a theoretical solution of the Sandström model for a very shallow fluid layer.

Consider a coordinate system with z directed vertically upward, x horizontal, the origin at the free surface of a horizontal liquid layer. The bottom of the liquid layer is at $z = -h$.

The steady-state equation of heat transfer is

$$u\,\partial\theta/\partial x + w\,\partial\theta/\partial z = \gamma\nabla^2\theta$$

where u and w are the component velocities, γ is the thermal diffusivity, and θ is the temperature. The steady-state equations of laminar motion are

$$\rho\,(u\,\partial u/\partial x + w\,\partial u/\partial z) = -\partial p/\partial x + \mu\nabla^2 u$$

$$\rho\,(u\,\partial w/\partial x + w\,\partial w/\partial z) = -\,\partial p/\partial z + \mu\nabla^2 w - g\rho$$

where ρ is the density, μ the dynamic viscosity, p the pressure, g the acceleration due to gravity. The steady-state equation of continuity is

$$\partial(\rho u)/\partial x + \partial(\rho w)/\partial z = 0$$

The equation of state of a real fluid is complicated, but we may introduce the following simple law which must be true over a small range of temperature

$$\rho = \rho_0\,(1 - \alpha\theta)$$

where x is the coefficient of thermal expansion. The free surface $z = 0$ is maintained at the temperature $\theta = t\cos \ell x$.

We suppose that $h \ll \pi/\ell$, that is the depth of the basin is much smaller than the wave length of the surface temperature distribution, and introduce the stream function ψ

$$u = \partial\psi/\partial z \qquad\qquad w = -\,\partial\psi/\partial x$$

the substitution $\beta = \rho_0 g/\mu$, and the transformation $v = -z/h$. We express both θ and ψ in powers of α [OBERBECK, 1879].

$$\theta = \theta_0 + \alpha\theta_1 + \alpha^2\theta_2 + \alpha^3\theta_3 + \ldots$$

$$\psi = \alpha\psi_1 + \alpha^2\psi_2 + \alpha^3\psi_3 + \ldots$$

We are thus led to an infinite set of equations determining the coefficients θ_1 and ψ_1

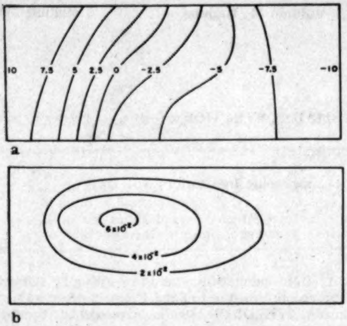

Fig. 1--(a) Temperature field, and (b) stream function field, for the numerical example described in the text

$$\partial^2 \theta_0 / \partial v^2 = 0 \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$\partial^4 \psi_1 / \partial v^4 = \beta h^4 \, \partial \theta_0 / \partial x \quad \ldots\ldots\ldots \quad (2)$$

$$(\gamma/h) \partial^2 \theta_1 / \partial v^2 = -(\partial \psi_1 / \partial v)(\partial \theta_0 / \partial x) + (\partial \psi_1 / \partial x)(\partial \theta_0 / \partial x) \quad \ldots\ldots\ldots\ldots \quad (3)$$

$$\partial^4 \psi_2 / \partial v^4 = \beta h^4 \, \partial \theta_1 / \partial x \quad \ldots\ldots\ldots \quad (4)$$

The inertia terms have been omitted from (4).

The boundary conditions are that at the surface θ_o must equal the maintained surface temperature $\theta_o = t \cos \ell x$, whereas $\theta_1, \theta_2 \ldots$ vanish at the top surface. At the bottom there is no vertical flow of heat

$$\partial \theta_o / \partial v = \partial \theta_1 / \partial v = \partial \theta_2 / \partial v = \ldots = 0$$

The boundary conditions upon the ψ-terms are that the surface and bottom are streamlines, that is $\psi_1 = \psi_2 = \psi_3 = \ldots = 0$ at $v = 0$ and $v = 1$.

The evaluation of these series has been carried out to two terms for the temperature and two in the stream-function

$$\theta = t\,(\cos \ell x - \sigma A \sin^2 \ell x)$$

$$\psi = -(\gamma/\ell h)\,(\sigma B \sin \ell x + \sigma^2 C \sin 2 \ell x)$$

where σ is a non-dimensional quantity defined by

$$\sigma = \alpha \beta t \ell^2 h^5 / \gamma$$

and A, B, C, are given by polynomials in v.

$$A = (10v^2 - 15v^4 + 8v^5)/960$$

$$B = (v - 3v^3 + 2v^4)/48$$

$$C = (165v - 295v^3 + 168v^6 - 54v^8 + 16v^9)/5{,}806{,}080$$

For purposes of illustration we may work out a numerical example. Suppose we take $t = 10°C$, $\mu = 0.01$ gm/cm sec, $\gamma = 0.001$ sq cm/sec, $\alpha = 2 \times 10^{-4}$/degree, $g = 980$ cm/sec · sec, $h = 1.36$ cm, $\pi/\ell = 210$ cm. Under these circumstances $\sigma \cong 200$. The series is only slowly convergent. The temperature and stream function fields are drawn in Figure 1. The principal interest of the example is the asymmetry of the temperature and velocity fields.

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MODEL STUDY OF SAND TRANSPORT ALONG AN INFINITELY LONG, STRAIGHT BEACH

Thorndike Saville, Jr.

Abstract--The rate of littoral transport of sand induced by wave action on a straight beach inclined to the direction of the wave crest was studied in a model and related to wave characteristics.

Experimental equipment--In a model basin approximately 66 ft by 122 ft in plan and two feet in depth, a wave machine and a model beach were arranged as shown in Figure 1. The wave machine was of the flap type in which the period of the waves could be varied by changing the speed of the driving motor, and the wave energy could be changed by adjusting the throw on the crank arms connected to the wave flaps. The wave height and wave length were determined by the period and energy settings.

The beach consisted of a six-foot strip of sand, covering the area of transport, and held in place laterally by screed boards (at the shoreward end) and a concrete pavement (seaward) (see Fig. 1). The sand beach was molded initially to a 1/10 slope by screeding, and then allowed to develop its profile under the action of the waves. The test sand was fresh water-washed, steam-dried, Del Monte sand. The mechanical analysis for the test sand is shown in Figure 2a. Its median diameter was 0.30 mm.

A sand trap and weighing device for recording the rate of littoral sand transport induced by direct wave action and by the littoral current was installed at the downbeach end (see Fig. 1). This device consisted of a metal hopper suspended on a beam which rested on a fulcrum. On one end of the beam a counterweight was attached (for zero setting), and the other end of the beam rested on a sylphon bellows which was liquid-filled and connected by tubing to a clock-driven Bourdon-type pressure recorder. The recorder was calibrated by placing known weights into the sand hopper. The efficiency of the sand trap was determined under operating conditions by feeding known weights of sand into the flow at the entrance to the unit. These latter tests showed that approximately 99 per cent of the pit run sand was deposited in the trap.

To insure that all sand transported along the beach would be carried into the weighing device, it was necessary to induce a flow into the basin containing the hopper. This was accomplished by the pumping arrangement shown in Figure 1. This pumping action was necessary in order to prevent the littoral current, upon reaching the trap, from flowing seaward; since such a seaward current would oppose the waves, sand deposition would occur in this region, and thus progressively change the rate of transport into the trap. The flow from the pump was returned to the main basin through a channel of relatively large dimensions at a correspondingly low velocity, so that no large scale circulation was induced in the basin. Thus, the length of beach studied approximated a section of beach of infinite length.

The relative amounts of littoral transport moving as suspended and bed load, as well as the distribution of bed-load transportation seaward from the berm, were measured by a modification of the slot-type samplers used in bed load measurements in natural streams [UNIVERSITY OF IOWA, 1940]. A series of four hopper-shaped, narrow boxes was buried in the beach, connected end to end, and extended seaward from the point of maximum uprush. The tops of the hoppers were open and at the same elevation as the beach floor. All sediment moving as bed load in a strip of beach equivalent to the length of the hopper settled into the hopper upon reaching its edge, and then was pumped continuously to an airtight settling bottle, where the sand settled out. The water was continuously pumped from the settling bottle, through the pump, back into the hopper. Thus, though there was a continual circulation of water and sand leaving the hopper, and water returning to the hopper, no current or eddies caused by the pumping action, developed around the hoppers, and the beach upcoast from the hoppers behaved exactly as though the hoppers were not in place. The rate and total amount of sand caught in each trap were determined by periodic weighing of the settling bottles.

Test procedure--All tests were conducted on beaches set at an angle of ten degrees with the

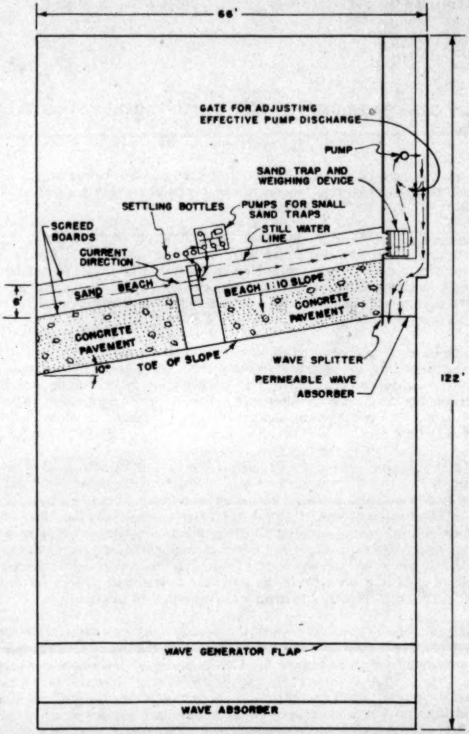
66'
GATE FOR ADJUSTING
EFFECTIVE PUMP DISCHARGE
PUMP
SAND TRAP AND
WEIGHING DEVICE
SETTLING BOTTLES
PUMPS FOR SMALL
SAND TRAPS
STILL WATER
LINE
SCREED
BOARDS
CURRENT
DIRECTION
SAND BEACH
BEACH 1:10 SLOPE
CONCRETE
PAVEMENT
CONCRETE
PAVEMENT
6'
10°
TOE OF SLOPE
WAVE SPLITTER
PERMEABLE WAVE
ABSORBER
122'
WAVE GENERATOR FLAP
WAVE ABSORBER

Fig. 1--Plan of wave tank showing relative locations of apparatus

wave flap, and consisting initially of an inclined plane molded to a 1/10 slope. The range of the principal wave characteristics is given in Table 1.

At frequent intervals during operation, measurements were made of the height and period of the waves. The period was measured by timing the motion of the wave machine, and the heights were obtained by point-gage readings at the trough and crest of the waves. The water depth generally was less than one-half the wave length; hence, the generated waves usually were shallow-water waves. The deep-water wave characteristics were computed from the wave characteristics at this depth by using curves showing the theoretical change in wave characteristics from deep water as a function of the depth-length ratio (d/L_o) (in which the subscript refers to deep-water condition) [WIEGEL, 1948]. No effects of tide variation were studied.

As used below, the terms upcoast and upbeach refer to the direction from which the littoral current comes. The terms downcoast and downbeach then would designate the direction in which the littoral current is moving.

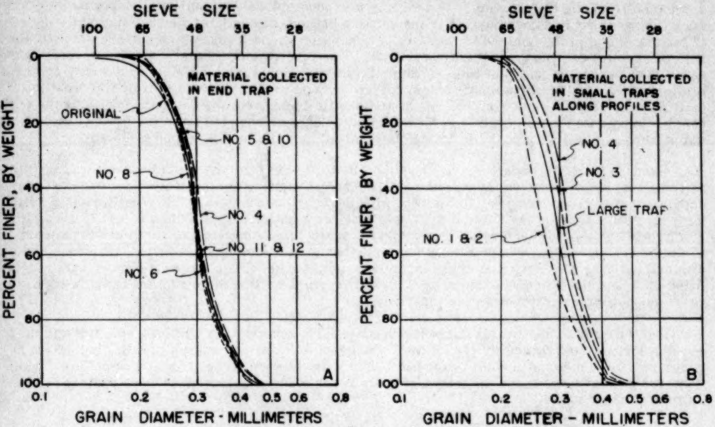

Fig. 2--Mechanical analyses of original sand, and of sand from large trap

Table 1--Summary of test conditions and results

Run number	Period T	Length L_o	Height H_o	H_o/L_o	Energy per wave E	Power P	Deep-water wave-angle α	Sand movement, dry weight	Longshore current
	sec	ft	ft		ft lb/ft	ft lb/sec ft	°	lb/hr	ft/sec
6	0.714	2.81	0.146	0.0523	0.468	0.315	10.0	23.3	0.319
7	0.846	3.67	0.129	0.0352	0.473	0.279	10.2	40.2	0.270
4	0.939	4.51	0.116	0.0257	0.469	0.251	10.5	62.6	0.254
5	0.996	5.08	0.110	0.0216	0.478	0.240	10.8	56.8	0.205
9	0.744	2.83	0.169	0.0597	0.611	0.411	10.0	29.9	0.398
8	0.845	3.65	0.147	0.0404	0.612	0.362	10.2	48.7	0.322
10	0.990	5.02	0.126	0.0252	0.616	0.311	10.7	88.2	0.241
11	1.17	7.01	0.106	0.0151	0.615	0.265	11.4	85.0	0.066
12	1.50	11.52	0.082	0.0071	0.605	0.202	13.1	18.2	. . .

In order to maintain as closely as possible the representation of a typical portion of an infinitely long straight beach, sand was introduced at the upcoast end of the beach at a rate as nearly possible as that determined from the rate of deposition in the large sand trap at the downcoast end of the beach. Hence, the amount of sand being moved in any unit length of beach at any time was the same the entire length of beach.

The large sand trap at the downcoast end of the experimental beach represented the coast beyond the point of its installation. This trap was responsible for the removal of all transported material from the current stream. After a run, the material deposited in this trap was removed and placed on the stockpile at the upcoast end of the beach, whence it was fed back into the zone of wave action as described above.

It also was necessary to introduce sand immediately downcoast from the linked system of small traps which romoved the bed load from the littoral drift. Here, the same sand that was removed from the drift was fed back onto the beach over a length of about three feet immediately downcoast from the trap system. The feeding was accomplished by manually emptying into this area the sand that had accumulated in the settling bottles, having first weighed it. Again the rate of feeding was that determined by the rate of deposition in the traps, which was the same as the rate of removal of sand from the littoral drift. Thus, the rate of littoral drift was preserved the same over the entire length of the experimental beach, except for a small area directly adjacent to the downcoast side of the small traps, despite the continuous removal of material from the drift at these traps. The beach alignment and profile consequently were the same along the entire beach. In both instances of feeding, the distribution of sand seaward from the limit of uprush was maintained as closely as possible to that determined from the small trapping system.

Although the initial beach profile had a 1/10 slope, this straight profile was altered by the action of the incident waves, and eventually reached an equilibrium profile which, once established, underwent no further changes. Coincident with the beach reaching a stable, equilibrium profile, the rate of transport of the littoral drift also reached a uniform equilibrium rate. This equilibrium condition was attained relatively quickly, usually after about two or three hours of operation of the model. Each run was continued for a number of hours after equilibrium conditions had been attained in order to insure that equilibrium was maintained and a uniform transport rate reached. Although transport rates were measured for the entire time that the model operated in each run, only the equilibrium rates are given in Table 1.

The strength of the littoral current was obtained by introducing a $KMnO_4$ solution into the current stream, and timing its travel over a ten-foot distance, measured parallel to the beach contours. Once the equilibrium beach had become established, vertical photographs were taken from the photographic crane directly above the beach. These photographs were analyzed to obtain the breaker angle.

After the experimental beach had operated for eight to 20 hr following attainment of equilibrium conditions, the model was shut down. At this point profiles of the equilibrium beach maintained by the incident waves were obtained by measuring the depth below still-water level with a movable point gage suspended from a removable beam. In order to obtain the average equilibrium profile and any possible change along the beach, profile measurements were taken at two different points: one at a distance eight feet upcoast from the small sand traps, and the other at a distance ten feet downcoast from the traps. To determine the effect on the beach profile and the distortion of the beach adjacent to the small sand traps, if any, due to the presence of the traps in the beach, another profile was taken at a point a few inches upcoast from the traps. This was compared with the equilibrium profile (see Fig. 3 and 4). Such an effect, if present, might influence the relative distribution of the transport as obtained from the small traps. All profiles were taken perpendicular to the beach contours. Alongshore photographs of the beach were taken with the model operating, with no wave action, and with the water drained out of the basin, to give the overall appearance of the equilibrium beach for each run.

At the end of most runs a mechanical analysis was made of the sand deposited in the large trap at the downcoast end of the beach. For several runs, in addition, analyses were made on the sand caught in each of the settling bottles into which the four small sand traps discharged.

After each run was completed, the beach sand was remixed to eliminate any effect of sorting from the previous run, and the beach then screeded smooth again and remolded to its original 1/10 slope. Another run was started, keeping waves of the same energy, but changing the period and height. Several runs were made at each constant energy setting of the wave generator, varying the period and height of the waves; then another series was made with increased energy content per wave.

General observations--Throughout the operation of the model, two types of sediment transport were observed; one, a zigzag longshore transport of bedload debris on the beach slope itself, the beach drifting of JOHNSON [1938], which is directly traceable to the wave action; and the other, a transport of suspended material by the longshore current, which is an indirect product of the wave action. Though both types are present over the entire beach slope, the former predominates along the foreshore of the beach; the latter, in the breaker zone along the offshore bar.

Sand transport--Of the two modes of transport (suspension and beach drift), it was found that the relative amount of material transported in suspension by the littoral current is a maximum, about 60 per cent of the total transport, under the action of the steepest, or storm, waves. It is a minimum under the action of the long, low, summer swell where it represents but a few per cent of the total transport (see Fig. 5a). Figure 5a shows the approximate relation of the two types of transport to the wave steepness, as the material caught in the small traps is primarily bed-load

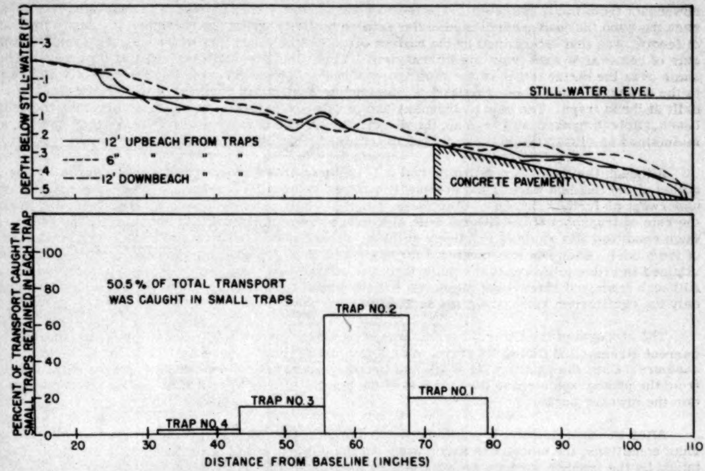

Fig. 3--Beach profiles and beach drift with steep waves, $H_0/L_0 = 0.0597$

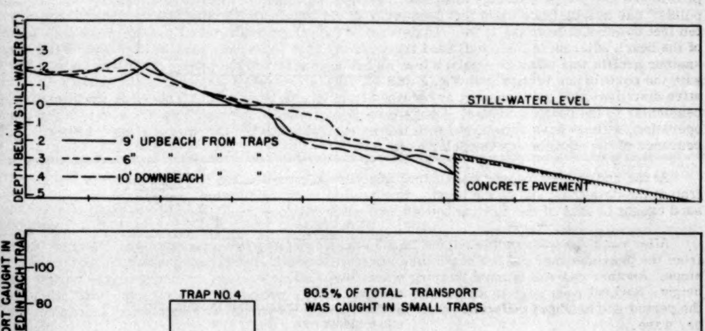

Fig. 4--Beach profiles and beach drift with long, low waves, $H_0/L_0 = 0.0151$

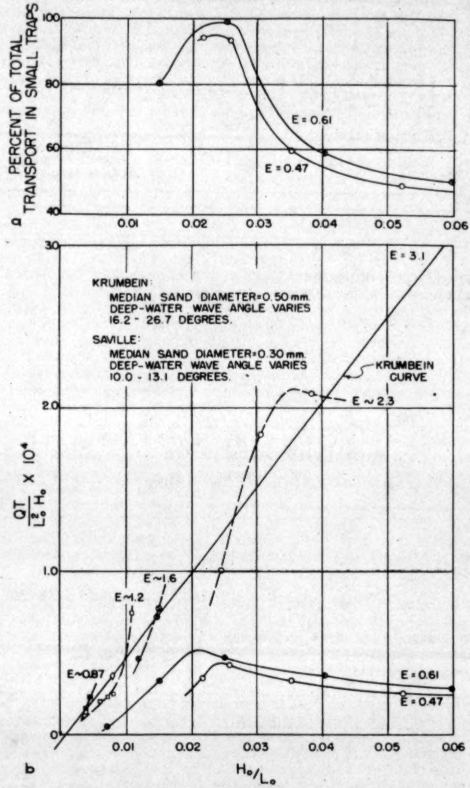

Fig. 5--Relationship between littoral transport of sand and wave steepness

material moved by the beach-drift mechanism. It may be noted that for steepness ratios less than 0.025, the relative amount caught in the small traps decreased. From this it might be inferred that the amount in suspension and available for transport by the longshore current had increased. Such, however, was not the case. The relative amount transported as suspended material did, in fact, increase, but this transport was by the mechanism of beach drift along the foreshore, and not by that of the littoral current (which was almost nil). The transport by beach drifting consisted almost entirely of bedload transportation for values of the steepness ratio greater than 0.025. As the H_O/L_O value decreases from this value, the relative amount of beach drift as suspended material increases sharply.

At an H_O/L_O value of 0.007, a large amount of material (127 per cent of the total transport) was caught in the small traps due to backfeeding from the downbeach side. The waves acting on this beach were the flattest tested, and refracted around almost parallel to the beach. There was no definite breaking of the wave, but rather a steepening, and then a sloshing or surging of the wave up the forebeach. The swash of the wave had a very definite alternation in direction, one wave setting a mass of water in motion in a downbeach direction, and the next in an upbeach direction. The component of motion in an upbeach direction was less than that in a downbeach direction, and a net transport downbeach occurred. This upbeach component, however, resulted in material being carried into the small traps from the downbeach side. Thus, more material was caught in the small traps than in the large trap at the downcoast end of the beach. This alternation in direction of motion of the wave swash was not peculiar to the trapping area, but was apparent along the entire beach.

The change from the littoral current transport predominant on storm beaches to the beach drifting transport predominant on summer beaches does not appear to be a gradual one, but it occurs over a very small range of steepness values, the critical value appearing to be H_0/L_0 equal to 0.03 (see Fig. 5a).

A similar critical value of wave steepness was noticed as a factor in the total rate of littoral drift. The amount of transport increased continually, at an increasing rate as the waves became less steep, reaching a peak at H_0/L_0 values of 0.02 to 0.025 (see Fig. 5b). As the waves then were made progressively less steep, the transport rate decreased rapidly from this peak. The peak value was about three times that obtained for the very steep waves (for the same energy per wave). This is a rather amazing result, as it might be expected that the most important agent in transporting the littoral drift would be the littoral current and that, therefore, the greatest amount of transport would occur when the combined effect of littoral current and amount thrown into suspension was the greatest. As both the strength of the littoral current and the amount of suspended material increase as the waves become steeper, it would then seem logical to expect the total amount transported to increase as the waves were steepened. Exactly the reverse was the case.

These experiments tend to show then, that it is not the action of the littoral current that is important in increasing the amount of littoral transport, but that of the waves themselves upon the forebeach, directly causing beach drift, whether as bed-load or suspended material. This would seem evidence that, contrary to popular belief, the steep storm waves of the winter season account for only a small proportion of the total drift. These storm waves erode away large amounts of beach material, but this material is not transported alongshore as littoral drift, it is deposited almost wholly at greater depths seaward. It is, rather, the intermediate waves and summer waves of steepness about 0.015 to 0.025 that do the major portion of transporting material along the shoreline.

It was noticed also that the transport occurred almost entirely in the surf zone and forebeach area. Though a great deal of material was in motion in the offshore zone, its movement was predominately perpendicular to the beach rather than along the shore. A tremendous amount of material in this area was moved, as can be shown by the beach profiles, but the movement was inshore or offshore, and not alongshore.

In extending these experimental results to prototype conditions, it must be remembered that these results were obtained on so-called equilibrium beaches, which were completely at equilibrium with the waves acting upon them. There was, therefore, no net transport of material perpendicular to the beach contours. Such a condition seldom, if ever, exists in nature. During the action of storm waves transport away from the beach occurs, the foreshore being eroded and carried out toward sea. While in the process of moving seaward, this material may well be made available for transport by the littoral current in addition to the material placed in suspension by the breaking wave. This may add considerably to the rate of transport along a natural beach under attack by storm waves. On the other hand, when the long, low, summer waves act on a beach, they bring material shoreward from the larger depths, building up and steepening the beach. Here, then, the waves would be acting on a forebeach that was of a gentler slope than the equilibrium profile. Hence, the gravitational effect that is most important in the process of beach drifting would not present as great an influence as on the equilibrium beach. The rate of transport might then be lower than that along a beach in equilibrium.

The rate of transport before equilibrium conditions were reached was considerably greater in runs 6, 8, and 9 than that obtained along the equilibrium beach. These three runs were with steep waves acting on the beach, and resulted in storm beaches. In runs 4, 5, 7, and 10 the transport rate while the beach profile was developing was about the same as the equilibrium rate. These four runs would be classified as intermediate and summer waves. In run 11 the initial transport rate was considerably less than the equilibrium rate. This run was with the beach under the action of one of the flattest waves tested. For the flattest wave, however, in run 12, the initial transport rate was somewhat higher than the equilibrium rate.

Due to this possible increase of transport over the equilibrium rate along a natural beach under storm-wave action, with a corresponding decrease along a beach acted on by summer waves, it might be possible for a greater rate of transport to occur with storm waves than with summer waves along a natural beach. However, whenever the same type of waves acts on a beach for a long period of time, an equilibrium condition is approached. Under these conditions the transport should behave as in the model tests, even though a true equilibrium beach does not develop. Such a condition might be expected to exist, for example, along the southern California coast during the late summer.

It also may be noted that though the littoral current in the model tests was not enough to set material in suspension, it may be along a natural coast, where currents of several knots are common. This again would tend to raise the transport rate along a natural beach acted upon by storm waves above that which might be expected from the model tests. However, it is doubtful whether this would have any great effect, and the model test results might be expected to apply at least qualitatively.

It must be remembered, also, in scaling these results up to prototype conditions, that strictly speaking only a completely undistorted model applies to wave studies. Hence, for average waves in the prototype, the beach material would correspond to gravel rather than to sand. Whether a very large reduction in the size of the model sand would result in different relationships is something that must be studied in the future.

Comparison with Krumbein's data--The important previous work on this subject was done by KRUMBEIN [1944] in some model work at Berkeley on a Guggenheim Fellowship. The rate of littoral transport was measured by feeding sand into a hopper at the upcoast end of the beach, the rate being determined by the ability of the waves to move it. The rate at which unit volumes of sand were fed into the hopper to maintain a maximum rate of removal was observed, and expressed as discharge units. The experimental data scattered somewhat, but plotted as a straight line function of the dimensionless variables $Q/(L_o^2 H_o/T)$ and H_o/L_o, where Q is the transport rate and L_o, H_o, and T are the deep-water wave length, height, and period, respectively. This shows a linear relationship between the littoral transport and the wave power. An expression relating these two dimensionless quantities also may be derived by methods of dimensional analysis.

Figure 5b shows a comparison of the data obtained by Krumbein in his model studies at Berkeley with those of the experiments reported herein. The parameters are those used by Krumbein in his original report. The beach sand used by Krumbein had a normal size distribution curve, the extreme values being 4.0 mm and 0.088 mm, and the median size being 0.50 mm. The wave flaps were set at a constant angle of 15° to the model beach. All the generated waves were of the shallow water type, and the deep-water wave angle, α_o, was greater than this value, varying from 16.2° to 26.7° for individual runs. These may be compared to a median grain diameter of 0.30 mm and a deep-water wave angle variation of 10.0° to 13.1° for the experiments described in this report.

Krumbein's experiments were not made at constant energy settings. If curves are drawn through points of similar energy content (dotted lines in Fig. 5b), a portion of a family of seemingly parallel curves results. It may be that the scatter about the linear relationship obtained by Krumbein is due to the chance of obtaining small portions of a series of curves, which then appeared to fit a straight line curve. Such a hypothesis appears to be substantiated by the results obtained from the experiments reported herein.

Littoral currents--The current velocities observed in the model experiments generally showed an increase in strength as the waves were steepened. This is because the longer waves feel the bottom sooner, and refract more than the shorter period waves. For these long waves which refract more nearly parallel to the beach, the longshore component of the mass of water set in motion by the breaking wave is much less than that for the shorter, steeper waves, and thus produces a smaller littoral current. The current velocity, however, is also a function of wave energy, increasing as the wave energy in increased. Thus, in one instance in the model a steeper wave, having less energy, did not produce as strong a current as another, flatter wave containing more energy.

Using the momentum approach developed by PUTNAM, MUNK, and TRAYLOR [1949] for predicting the longshore current, an average value of the friction parameter, k, for the beach was determined from the observed velocities. This average value was k = 0.0135. It may be compared with that obtained for a natural beach at Oceanside of k = 0.0078, and for model tests of a "natural sand bonded with cement to prevent erosion" of k = 0.0397 [PUTNAM and Others, 1949]. Using the average value of k, the strength of the littoral current then was predicted from the momentum equation. A comparison of the predicted values of current with the observed values is shown in Figure 6. Despite large variations in the individually determined values of k (amounting to more than 100 per cent), the values predicted by using the average k value deviate by less than ten per cent from the observed over the greater portion of the range studied. At very low values of wave-steepness ratios, the momentum equation appears to no longer hold. The correlation above is all the more remarkable as the observed values are averages, with fluctuations of ± 25 per cent. Neither the average value nor the degree of fluctuation varied with position along the beach. The fluctuations were evidently not associated with position along the beach, but with time, and were probably due to the variation in wave height during the time interval. It is interesting to note that current fluctuations of ± 25 per cent also were obtained at Oceanside.

The maximum current observed was always along the offshore bar. Seaward from the bar, the current velocity dropped very rapidly, reducing to about one-fifth the value along the bar in a distance of only a few inches. The current velocity also dropped toward shore from the bar, but not as noticeably. The dye used in observing the current traveled at almost a uniform speed in the whole area from the bar to shore, but along the foreshore this was due more to the direct action of the wave in beach drifting than to the littoral current.

Beach profiles--The equilibrium profile of the beaches changed from that of a storm beach to that of a summer beach as the waves were made less steep. With steep waves, the slope flattens, the bar and trough become very pronounced (there may be several bars), and the beach contours become deeply curved (see Fig. 3). With long. low waves, the profile steepens, the bar and trough become less predominant and move inshore, and may disappear entirely, and the beach contours become straight and parallel (see Fig. 4). The change from a prograded to retrograded beach appeared to occur at a steepness value of 0.04. The bar disappeared for values of H_0/L_0 less than 0.02. These findings are in accordance with those of WATERS [1939]. No appreciable change in profile occurred along the beach except for a small area immediately downstream from the small-trap system. The waves, having dropped a great deal of their sediment into the traps, picked more sand up immediately below the traps, and tended to scour in that locality. Feeding of the sand accumulated in the settling bottles into this area enabled the beach to return to its equilibrium profile within a three-foot distance from the small traps. Downcoast from this point the equilibrium profile existed. In many of the runs the beach just upcoast from the traps tended to build out beyond that of the equilibrium position. While the profile itself was the same as the equilibrium, the entire profile was displaced seaward. This building out, while not noticeable to the eye when looking along the beach, is demonstrated in the profile comparisons. The tops of the traps were evidently placed slightly higher than the position of the equilibrium beach floor, and the beach built outward, and up to them. Equilibrium conditions did attain, however, and this slight beach building does not affect the validity of the results.

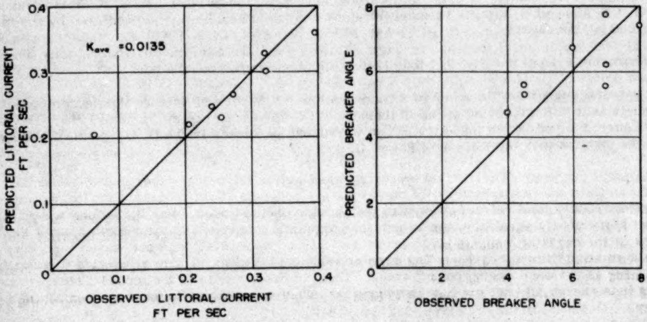

Fig. 6--Comparisons between observed and predicted values of littoral current and of breaker angle

Breaker angles--Figure 6 shows a comparison of the breaker angles predicted from theory with those obtained from vertical photographs. The scatter seems to be random, and the deviation may be ascribed to the difficulty in recognizing the initial point of break of the wave in the photographs.

Sand sorting--The mechanical analyses of the material transported along the beach and deposited in the large trap at the downcoast end of the beach show no appreciable change in grading from the original sand sample initially supplied to the beach (see Fig. 2). This material represents the total transport, whether by littoral current or by beach drifting. Hence, there was no differential grading of the sand longitudinally along the beach, regardless of the characteristics of the incident waves. This would seem to indicate that the size of the particle within the mixture does not influence the rate of transport, but that all particles are transported at the same rate along the beach. The findings of JONES [1948] and KRUMBEIN [1944] corroborate this conclusion.

Analyses also were made on the material deposited in the small sand traps for several of the runs. Results for a typical run are shown in Figure 2. It will be noticed that there was considerable grading of the material in a direction perpendicular to the beach contours, with the heavier material being carried along the foreshore, and the lighter material more at sea.

Summary--The following statements summarize the important conclusions which are pertinent to the experiments conducted. Strictly speaking, in the prototype these results correspond to gravel beaches which are in complete equilibrium with the waves acting upon them, and which may be considered as segments of infinitely long straight beaches.

(1) Two types of sediment transportation were observed in the model operation. Lateral translation, due to the direct oscillatory wave action, produced a zigzag movement, mainly of bed load particles. Longitudinal translation by the generated longshore current accounted for much of the suspended load movement.

(2) On equilibrium storm beaches, the sediment transportation was produced mainly by the movement of material in suspension by the littoral current. On equilibrium ordinary or summer beaches, the transport of sand was almost entirely by the mechanism of beach drifting caused by the direct action of the waves. The transition between the predominance of these different types of transport was a sudden one, occurring at a steepness ratio of about 0.03.

(3) The transport along summer beaches was much greater than that along storm beaches for waves with the wame energy content. The peak transport occurs at a steepness ratio of from 0.02 to 0.025, and is almost entirely by beach drifting.

(4) The transport rate drops very rapidly for steepness ratios of less than 0.02.

(5) The transport rate increased with an increase in wave energy; other conditions remaining the same.

(6) The transport occurred almost entirely in the area shoreward from the breaker.

(7) No appreciable sand sorting longitudinally along the beach occurred, regardless of wave characteristics. This indicates that particle size does not influence the amount or rate of transported material.

(8) Sand grading perpendicular to the beach did occur, the heavier particles moving inshore.

(9) The strength of the littoral current along a straight beach may be predicted by the momentum relationships obtained by PUTNAM, MUNK, and TRAYLOR [1949] with reasonable accuracy.

(10) The maximum littoral current was obtained along the bar, the current velocity decreasing sharply seaward of the bar, and less noticeably shoreward from the bar.

Acknowledgments--This paper is a condensation of a Master of Science thesis completed in the Department of Engineering at the University of California, Berkeley, in June 1949. The work formed part of an extensive investigation on waves and surf made in the Department of Engineering in cooperation with the Bureau of Ships, U. S. Navy.

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THE SOURCES OF HEAT AND WATER SUPPLY OF THERMAL SPRINGS, WITH PARTICULAR REFERENCE TO STEAMBOAT SPRINGS, NEVADA

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Abstract--Thermal springs are classified as: (1) non-volcanic (heat and water supply are not immediately related to volcanism); (2) intermediate (excess heat is due to abnormally high geothermal gradients resulting from volcanism, but the water is entirely meteoric); (3) volcanic (excess heat, much of the mineral content, and part of the water are from a volcanic source).

Lines of evidence indicating the origin of a thermal spring system are (1) temperature relations, including surface temperatures of springs, geothermal gradients of the spring system, and possible sources of excess heat; (2) mineral content of the thermal water and its possible sources, with attention focused on the more soluble components that are seldom deposited in veins (Chlorine, boron, and to a lesser extent sulphur and carbon dioxide in their various chemical combinations are critical indicators, particularly when considered together.); (3) composition of associated gases; (4) variations in spring discharges and water levels, as related to variations in meteoric water supply; (5) isotopic composition of spring and meteoric waters; and (6) regional and local geology, and in particular the age and extent of any volcanic rocks. These lines of evidence, when applied to Steamboat Springs and evaluated, definitely indicate a volcanic origin.

The three-dimensional picture of the Steamboat Springs system is described. Meteoric water migrates downward and toward the core of the system, with stratigraphic and structural control, and mixes with rising volcanic water. This migration is caused by differences in density related largely to temperature; artesian conditions are not essential.

Introduction--Interest in the geology and origin of thermal springs of the United States was first apparent only a little more than 100 years ago. Most of the literature on the subject concerns individual springs or groups of springs, but a few general reports have been published. The most recent and most inclusive of these is by STEARNS, STEARNS, and WARING [1937]. The reader is referred to this paper for a summary of the general problems and for an extensive bibliography on the thermal springs of the United States.

Major questions on the subject concern the sources of heat, water supply, and mineral content; in general, investigators have shown marked disagreement on criteria to be used, as well as upon their conclusions.

Geologists of the world have been interested in Steamboat Springs, Nevada, since 1888, when BECKER [1888] stated that mercury, antimony, gold, silver, copper, and other metals were being deposited from the thermal waters. The main purpose of the present investigations of the United States Geological Survey is to determine the origin of these thermal springs and any bearing they may have on processes of ore deposition. Preliminary results of these and other phases of the investigations have been published elsewhere [BRANNOCK and Others, 1948].

The springs are 19 km south of Reno, Nevada, and 11 km northwest of Virginia City and the famous Comstock Lode. Steamboat Creek, a north-flowing tributary of the Truckee River, flows along the east base of the principal spring terrace. The climate is semiarid and is strongly influenced by the Sierra Nevada. The annual precipitation is about 30 cm at the springs but increases westward to about 75 cm at the base of the Carson Range, a branch of the Sierra Nevada.

The principal springs and fumaroles occur in an area of about 2.5 sq km, at altitudes ranging from 1400 to 1500 m above sea level. Generally no more than about 12 springs have appreciable discharges at any one time, and these range from one to 75 liters/min, with a total measured discharge of 100 to 375 liters/min. The total flow of thermal saline water from the area is probably over 3000 liters/min, including about 925 liters/min discharged directly into the bed of Steamboat Creek and an estimated average of 1850 liters/min from the producing thermal wells.

In surface temperature, the springs range from about 45° to 95.5° C, the latter is the approximate boiling point of pure water at the altitude of the area.

Geology--The springs issue from the northeast base of Steamboat Hills, which lie near the center of a north-trending structural trough bounded on the east by the Virginia Range and on the west by the Carson Range. The oldest rocks of the region are metamorphosed volcanic and sedimentary strata, intruded by granodiorite related to the Sierra Nevada batholith. These rocks are overlain in many places by andesite of Tertiary age, but the granodiorite in the northeastern part of Steamboat Hills is immediately overlain by a basalt flow that is younger than the andesite. The youngest volcanic rocks of the region are extrusive domes of pumiceous rhyolite of Quaternary age. Deposition of alluvium and hot spring sinter, which began prior to the extrusion of the basalt and rhyolite, is still continuing.

The dominant structural features of the region are north-striking faults that to a large extent define the ranges. Parallel faults occur in alluvium of the trough and in Steamboat Hills. The fault zone localizing the springs is the part of this system that lies nearest the center of the trough. However, this zone appears at least in part to be a westward dipping thrust fault that probably developed in Quaternary time. Because of its topographic position and a relatively high water table, the fault zone is a favorable route of exit for subsurface water.

Sources of heat--The individual problems of the sources of heat, water supply, and mineral content are closely related, and each has a direct bearing on the others. The most likely sources of heat for thermal springs are generally considered to be [STEARNS and Others, 1937] (a) from the "normal" increase in temperature of the Earth with depth, (b) from an underlying body of hot or possibly molten rock, (c) from zones where there has been faulting of the rocks with resultant development of heat, (d) from chemical reactions beneath the surface, or (e) from the energy derived by disintegration of radioactive elements.

According to the theory of deriving heat from the "normal" or average regional increase in temperature with depth, the heat is transferred to the surface by means of deeply circulating meteoric water. VAN ORSTRAND [1939] has shown that the average temperature gradient (1° C/m) in the sedimentary areas of the United States is almost certainly less than 1° C per 32.9 m of depth and possibly as little as 1° C per 60.4 m.

Proponents of this "normal" geothermal gradient theory have commonly assumed that the water arrives at the surface with little or no decrease in temperature from its maximum, and that the depth of circulation is indicated by the temperature. Within such a spring system from the recharge area to the base of circulation, the downward movement of cold meteoric water must lower ground temperatures. The temperature gradient, therefore, must be less than that of the surrounding normal region. Because water at the base of circulation may never reach the temperature of the surrounding rocks, and because it reaches the surface at an appreciably higher temperature than that of the surrounding normal ground, the thermal gradient within this part of the spring system must also be less than that of the normal area outside of the system. It is concluded, therefore, that an outstanding characteristic of a system of thermal springs deriving its heat entirely from a normal temperature increase with depth is a lower gradient than that of the surrounding region.

The temperature relations existing in a region where local excess heat is available from molten or hot volcanic rocks is very different from that described above, and, in general, the temperature increase with depth is higher than normal.

Heat derived from friction due to movement of rocks on faults may be an important local or auxiliary source for thermal springs. However, in the western United States where numerous recent faults are known, there is little or no apparent correlation between the recency of movement and abundance or temperature of springs. The common association of thermal springs and faults is best explained on the basis of structural control rather than a genetic heat supply. DAY and ALLEN [1924] and others have generally concluded that known chemical reactions are not an adequate source for the heat of thermal springs. A number of investigators have found no apparent relation between the temperatures of springs and the degree of radioactivity of the issuing waters. Atomic disintegration therefore does not appear to be an important direct source of heat for thermal springs, although indirectly it may be a critical factor in maintaining or supplementing the internal heat of the Earth, and it may be one of the basic causes of volcanism.

Temperature gradient at Steamboat Springs--Measurements made in thermal wells at Steamboat Springs show very high temperatures at shallow depths. For example, the temperature at the

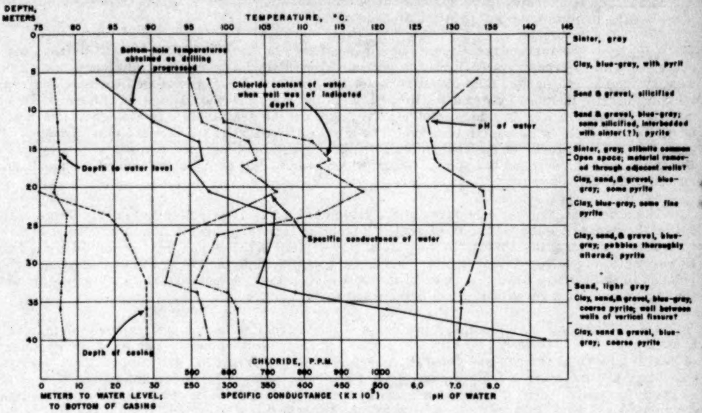

Fig. 1--Temperatures, chloride content, and other data of Steamboat Well no. 4, Steamboat Springs, Nevada

bottom of a well 13.2 m deep is 114° C, and the bottom of another 47.5 m deep is 138° C.

Steamboat Well no. 4 was drilled in 1947 in the southeastern part of the thermal area. The rate of temperature increase is very irregular (see Fig. 1), but within the explored zone the average is extremely high. The gradient can be computed by several different methods, depending on the assumed temperature at the surface, but no method is completely satisfactory. The results range from 1° C per 0.33 to 0.82 m of depth, or at least 40 times the normal increase. The very high temperature gradient at Steamboat Springs is believed to be very strong evidence that the springs derive their heat from a volcanic source.

Composition of the gases--The gases evolved from the springs, excluding water vapor, contain 95 to 99 per cent of CO_2, and 0.1 to one per cent of H_2S; the remainder consists of nitrogen, argon, and oxygen. The atmospheric gases are probably derived in large part from the atmosphere and carried underground in solution in meteoric water, although there is little doubt that some nitrogen and argon are from a volcanic source. The gases are similar in many respects to those of other high-temperature springs, and to volcanic emanations.

Composition of the water--A typical analysis from Steamboat Springs is compared with those of other thermal waters of the United States (see Table 1). The first five analyses are believed to be typical of most thermal water discharged at or near boiling. In these, sodium and potassium are generally the dominant cations. The most abundant anion is generally chloride but this may be exceeded by carbonate, bicarbonate, or, rarely, borate or sulfate. Relatively high chlorine, boron, and sulphur, and a great dominance of total CO_2 (as carbonate, bicarbonate, and CO_2 gas) over calcium and magnesium are considered by the authors to be the outstanding chemical characteristics of high-temperature thermal springs, except in acid-sulfate springs. The common occurrence of these same components in volcanic fumaroles indicates that they are at least in part of volcanic origin, although supplementary sources such as saline deposits must always be considered.

Spring no. 33 of Norris Basin, Yellowstone Park, (see Table 1), an example of the acid-sulfate type, has received much attention from ALLEN and DAY [1935, pp. 65, 393-448]. The origin of this type of water has not yet been demonstrated conclusively. The springs are generally situated on relatively high ground and are characterized by meager discharges in comparison with the chloride springs. The authors of the present paper believe that the majority, if not all, of acid-sulfate springs are a superficial phase of thermal activity generally occurring where the water level of saline spring water is below rather than at the ground surface. Steam, it is believed, is evolved by boiling from the water table and is condensed at or near the surface when it comes in

Table 1--Composition of thermal waters of volcanic association

Ion or item	No. 21, Steamboat Springs, Nevada[a]	Sulphur Bank, Calif.[b]	Geyser, Beowawe, Nevada[c]	Old Faithful, Yellowstone[d]	Jupiter Mammoth, Yellowstone[e]	No. 33 Norris, Yellowstone[f]	Congress Pool Norris[f]
	ppm	ppm	ppm	ppm	ppm	ppm	ppm
H	. . .	. . .	. . .	. . .	. . .	0.60	3.00
Ca	8.7	56.3	2	4	338	4	16
Mg	1	57.1	none	. . .	71	2	3
Na	695	1389	239	372	143	48	458
K	74	68.1	33	31	56	30	108
Li	5	. . .	. . .	. . .	. . .	. . .	. . .
Al	<1	. . .	tr.(?)	none	tr	1	3
Fe	<0.1	tr.	tr.	tr.	none	3	8
NH_4	<1	543	4	. . .	. . .	342	21
CO_3	N.F.[g]	none	173	100	none	none	none
HCO_3	276	1891	129	none	871	none	none
SO_4	156	1984	97	21	528	1105	503
S_2O_3	14	. . .	1	tr.	tr.	none	none
S	N.F.[g]	. . .	. . .	. . .	. . .	. . .	none
Cl	921	489	47	435	169	5	634
F	2.1	. . .	11	22	. . .	. . .	. . .
AsO_4	4.3	none	none	. . .	. . .	0.9	. . .
SbO_3	0.3	none	. . .	. . .	. . .	. . .	. . .
B_2O_3	176	1700	23	18	. . .	8	. . .
SiO_2	262	67.4	449	357	46	303	509
Total	2596	8245	1208	1360	2222	1853	2263
°C	83.9	80.6	boiling	boiling	71.0	hot	hot
pH[h]	6.3	7 ±	9.5±	9.0±	6.5±	3±	2±

[a] Analyst, W. W. Brannock.
[b] Analyst, M. Fleischer.
[c] ALLEN and DAY [1935, p. 174]; analyst, E. T. Allen.
[d] ALLEN and DAY [1935, p. 249]; analyst, E. T. Allen.
[e] ALLEN and DAY [1935, p. 373]; analyst, E. T. Allen.
[f] ALLEN and DAY [1935, p. 469]; analyst, E. T. Allen.
[g] Looked for but not found.
[h] Determined or estimated from later measurements by Donald E. White.

contact with cooler ground or previously condensed water. In general the dissolved salts are left behind in the saline water but some intermixture may occur when the water level of the saline water rises, because of increased water supply, to the level of the acid spring. Some salts may also be carried as a spray with the steam. The acid-sulfate characteristics of the condensed steam are the result of atmospheric oxidation of H_2S that was evolved with the steam. It should also be mentioned here that where the supply of H_2S is abundant and discharge from the area is relatively low, the saline water itself can become acid and heavily charged with sulfate as a result of the same reaction (see analysis of Congress Pool, Table 1).

Warm Springs, Georgia, is one of the clearest examples of a thermal spring that probably owes its temperature to a deep circulation of meteoric water (see Table 2). Even here there may be a remote connection with volcanism, possibly in providing a local higher-than-normal temperature gradient, but evidence concerning the gradient is lacking. It is clear, however, that there is no significant contribution from a volcanic source in the water of Warm Springs. The content of chloride is approximately the same as that of surface waters of the region, and boron has not been detected in either type. Also, the excess of CO_2 over the amount required for calcium and magnesium carbonates is approximately the same in both waters. The analysis of water from the South Steamboat Well (see Table 2) is believed to represent an intermediate type, consisting essentially of meteoric water that has derived its excess heat from a volcanic source.

At Steamboat Springs the only available source for most of the chlorine, sulphur, boron, and excess carbon dioxide appears to be volcanic. The bed-rocks are notably low in carbonate and

Table 2--Comparison of surface waters with warm meteoric waters of Warm Springs, Georgia, and Steamboat Springs, Nevada

Ion or item	Cascade Branch SW of Warm Springs, Ga.[a]	Warm Springs, Georgia[b]	Steamboat Creek, Nevada[c]	S. Steamboat Well, Steamboat Springs[d]
	ppm	ppm	ppm	ppm
Ca	0.3	21	26	2.0
Mg	0.3	12	12	0.4
Na	1.4	1.6	21	69
K	0.2	3.6	6.4	6.8
Li	. . .	. . .	0.2	0.2
Fe	0.04	0.01	. . .	. . .
NO_3	0.05	0.10	N.F.[e]	N.F.
CO_3	none	none	N.F.[e]	19
HCO_3	2.0	118	192	146
SO_4	1.1	7.3	5.5	2.3
Cl	1.6	1.8	4.2	8.4
F	0.0	0.1	0.4	0.4
B_2O_3	none	none	N.F.[e]	0.5
SiO_2	4.5	23	48	4.7
Total solids (180°C)	12	121	208	182
pH	. . .	. . .	7.6	9.0
°C	8.9	31.1	10±	43

[a] HEWETT and CRICKMAY [1937, p. 18]; analyst, W. L. Lamar.

[b] HEWETT and CRICKMAY [1937, p. 17]; sample from East Source, collected in 1933; analyst, W. L. Lamar.

[c] Analyst, W. W. Brannock.

[d] On southernmost border of Steamboat Springs; analyst, W. W. Brannock.

[e] Looked for, but not found.

are not known to contain saline deposits within a radius of many kilometers, except for anhydrite and gypsum 18 km to the southeast. The valley fill of Truckee Meadows north of the springs could conceivably contain saline deposits because of the graben-like nature of the basin. This is highly unlikely, however, because the Truckee River has probably always provided external drainage and because nonthermal ground water of the basin contains little if any more chloride than the surface water. In addition the springs issue from granodiorite and alluvium near the edge of the valley fill. It is therefore highly improbable that any of the mineral content of the springs was derived from saline deposits or that appreciable CO_2 was obtained by reaction with carbonate rocks.

Rhyolite domes--The springs are on a line of possible structural weakness that connects three or more domes of rhyolite that may have been extruded after the oldest hot spring sinter was deposited. A structural and temporal connection with volcanism is indicated, and all other available evidence consistently points to volcanic rocks as the source of the heat and of much of the gas and mineral content of the springs.

Sources of water--Since June 1945 systematic measurements have been made of spring discharges, temperatures, and mineral content. The purpose of these measurements was to determine the effects of rainfall, and other changes in available water supply. In general, measurements were made one or two times a week, but daily observations were made througout most of the interval of November 4 to December 30, 1946 (see Fig. 2). Results of similar measurements for the first eight months of the investigation are described elsewhere [BRANNOCK and Others, 1948, pp. 216-219]. Figure 2 indicates that the springs are not influenced by a slight precipitation but that a heavy rain has marked effects. The fact that the response is almost immediate, increasing discharge and decreasing chloride concentration and temperature, indicates that the effects are largely superficial in character and are due to an intermixture of rainwater and thermal saline water in the uppermost part of the spring system. Probably only a part of this rainwater falls on the spring terrace; the balance is believed to come from the small drainage basin immediately to the west.

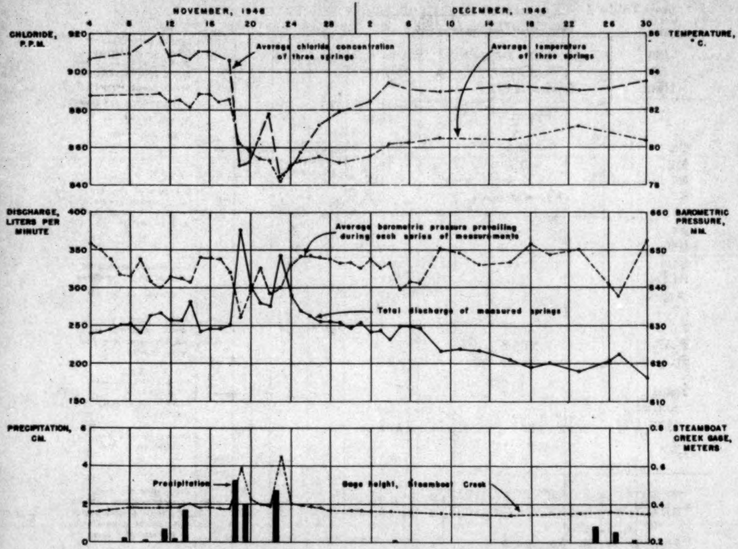

Fig. 2--Total measured discharge from hot springs compared with related factors for period of November 4 to December 30, 1946, Steamboat Springs, Nevada

The influence of a heavy rain on the discharge is largely terminated within a few days. The chloride curve, it is believed, reveals a deeper influence. The high point in concentration on December 4, 1946, follows the previous rain by 11 days. The record of June 1945 through December 1946, previously referred to, contains similar anomalous changes in the chloride curve at intervals of about eight or ten days after the two major storms of the period. The general rise in chloride from November 23 to December 4 is due to a gradual flushing-out of rain water that fell directly on the thermal area. The authors believe that the increased supply of meteoric water in the recharge area following each rain was eventually effective in deeper parts of the spring system in increasing the proportion of meteoric water mixing with highly saline water. The lag of eight to 11 days includes not only the time necessary to make effective the change in mixture (due to slow transmission of hydrostatic pressure in sediments of low permeability) but also the time required for the new mixture to reach the surface.

The records show that the discharge of individual springs as well as of the group as a whole is influenced by many factors. In addition to those already discussed, barometric pressure is a major short-term factor; the springs tend to behave as inverted water barometers with efficiencies ranging from zero to nearly 100 per cent. Influence of Earth tides has been detected in some water level fluctuations but this is a minor factor. One of the outstanding characteristics of most individual springs is their ever-changing behavior. Many have stopped flowing and new as well as old vents have begun to flow. Most of these changes are probably due to the filling or clearing of subsurface channels, but in addition, some major fluctuations have occurred for which there is no apparent explanation. It is possible that the volume of volcanic water emitted from its source is variable rather than constant, as is generally assumed.

Slight differences are indicated in the isotope ratios of O^{16} and O^{18} in water samples from Steamboat Creek and Spring no. 27, Steamboat Springs. In 1946 Malcolm Dole of Northwestern University determined the densities of purified parts of samples by precise methods that he had developed [DOLE, 1936; SWARTOUT and DOLE, 1939; DOLE and SLOBOD, 1940]. The results

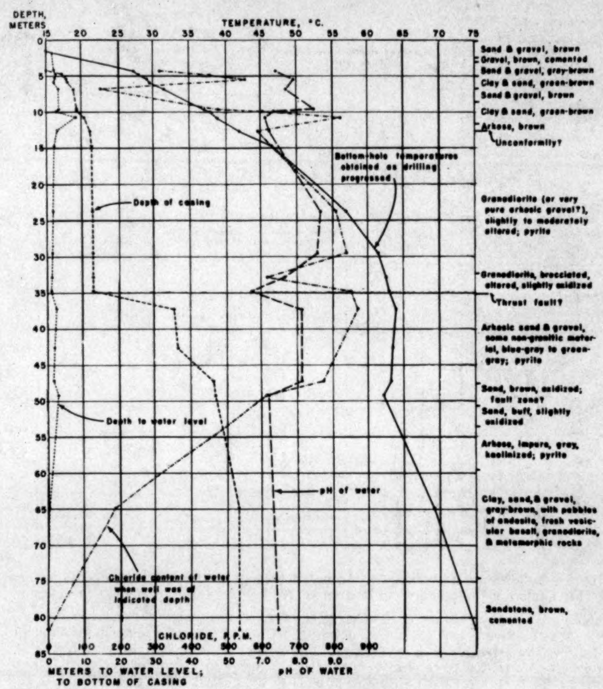

Fig. 3--Temperatures, chloride content, and other data of South Steamboat well, Steamboat Springs, Nevada

include effects of both oxygen and hydrogen isotopes, and are expressed in terms of water density in ppm relative to Lake Michigan water. The differences in density between Steamboat Spring and Steamboat Creek waters due to isotopic differences, relative to Lake Michigan water are: Steamboat Creek water, -0.7 ppm; and Spring no. 27, Steamboat Springs, -0.55 ppm.

Dole submitted the remaining part of these same samples to Harold C. Urey of the University of Chicago in 1948. Urey determined the ratios of the oxygen isotopes in these samples by mass spectrograph [UREY, 1947], and the results were computed in terms of density differences by Dole as follows: Steamboat Creek water, -1.75 ppm; and Spring no. 27, Steamboat Springs, -1.22 ppm.

The differences between the two sets of determinations may be due at least in part to differences in the hydrogen isotope ratios of the two waters.

The percentage of volcanic water in spring no. 27 can be computed if the following assumptions are made: (1) Atlantic Ocean water is 1.6 ppm heavier than Lake Michigan water, due to a higher content of O^{16} as a result of isotopic fractionation [DOLE, 1936]; (2) Atlantic Ocean water is representative of the total volcanic water that has separated from igneous rocks in the past; (3) volcanic water contributed to Steamboat Springs is similar in its ratio of oxygen isotopes to the total of all volcanic water; (4) all meteoric water in the spring system is isotopically similar to the analyzed sample from Steamboat Creek; and (5) selective interchange if isotopes has not occurred between spring water and minerals of rocks through which the water has migrated. It should be emphasized that a very extensive program of research must be completed before most of these assumptions can be considered reliable. In addition, repeat determinations should be made on the spring and creek waters to determine the reliability of the indicated differences.

On the basis of these assumptions, Dole's results indicate that about six per cent of the spring water is volcanic in origin; Urey's results indicate that about 16 per cent is of volcanic origin.

Three-dimensional picture of upper part of the spring system--In general the highest concentrations of chloride and probably of boron are found near the center of the area of active springs, where the temperatures are also generally higher. The picture in relation to depth remained obscure until several wells were drilled for hot water in 1946-1947. Steamboat Well no. 4 (see Fig. 1) was drilled in the southeastern part of the thermal area, where very high temperatures were found at relatively shallow depths. The South Steamboat Well is 250 m farther to the south near the extreme southern edge of the thermal area. The data for this well are shown in Figure 3. The outstanding features are (1) temperatures and geothermal gradients are low, compared to Steamboat Well no. 4; (2) a major zone of saline water extends from the water level to a depth of about 50 m below the surface; (3) horizons of less-saline water occur within this major zone; and (4) water below about 38 m decreases in salinity with depth. The well is cased to 53 m, and water below this depth is almost free of dissolved mineral matter (see Table 2). Since the completion of the well, its chloride content has been about eight ppm. For comparison, Steamboat Creek water contains four to six ppm of chloride and average spring water about 900 ppm. The bottom zone in this well is therefore essentially water of meteoric origin. It should be mentioned in regard to Figures 1 and 3 that observations and water samples were obtained from these wells at least 12 hours after the last previous drilling. The data are arbitrarily referred to the depth existing in the well during each series of measurements, although they are actually resultants of unknown combinations of conditions existing between the bottom of the casing and the bottom of the well.

South Steamboat Well penetrated sand and gravel from the surface to about 12 1/2 m, and from this depth to about 35 m penetrated what is believed to be granodiorite, in part thoroughly altered and shattered.

The remainder of the well is in arkose and cemented sandstone that is unlike any bedrock that crops out in the area. It is believed to be Quaternary valley fill, over which granodiorite has been thrust from the west, the fault being a part of the Steamboat Springs fault zone. If these relations are typical of the fringes of the spring system, they indicate a border zone strongly controlled by one or more faults. Near the surface, within at least a part of the border zone, saline spring water is migrating outward, roughly horizontally, and mixes with a small amount of less saline water that may be moving inward. Below this, a major zone of essentially meteoric water is migrating into the spring system and is being heated, in small part from above but largely from below. The movement of water is controlled by faults as well as permeable strata, and true artesian conditions are probably absent. The same fault or fault zone may be utilized by both the up-flowing saline water and down-flowing meteoric water, and is comparable in many ways to a huge convection system; the central part of the zone is extremely hot, so the average density of the water is relatively low in spite of its salinity; laterally from the central core but still within the fault system and adjacent strata, meteoric water is migrating inward and downward. Its average density is perhaps five to ten per cent greater than that of the saline water because of its low temperature. Such a system as this is adequate to explain the facts; artesian conditions, if present within the valley fill, would facilitate the underground movement of water, but they do not appear to be essential.

Fluctuations in water level of the South Steamboat well, shown by records from an automatic water-stage recorder, prove that the meteoric water of this well is definitely influenced by the saline Steamboat wells. When producing, the latter may be considered as more-or-less continuously erupting geysers. When the Steamboat wells are shut down, the water level, of the South Steamboat well responds in less than one-half hour, and begins to rise. A reverse effect occurs when the Steamboat wells are again producing.

If the three-dimensional picture of the system that has been described is correct, the saline water of the central core becomes more and more saline with depth, and temperatures are in general closely related to the salinity of the water (see Fig. 1 and 3). The evidence supports the belief that a relatively small amount of highly saline water is being evolved from extremely hot volcanic material.

Sources of mineral content--Most of the evidence bearing on the sources of the mineral content of the thermal water has already been presented. In these particular springs, all or most of the chlorine, boron, sulphur, CO_2, arsenic, antimony, and fluorine is believed to be of magmatic origin. The source of silica and the common metals is not as evident; some may be of volcanic origin and some derived by solution from rocks adjacent to the channels of migration. It is highly probable that, if the volcanic water was originally rich in the alkali metals, much of the potassium originally in the water has been substituted for sodium in the wall rocks by means of base exchange. The absence of ammonia is notable, in contrast to the abundant ammonia in some other springs. Its very sporadic occurrence in thermal water suggests that, when present, it may come from sedimentary rocks containing organic material and that, when absent, the waters do not traverse sediments with an appreciable organic content.

Application of principles in other areas--Commonly, high-temperature springs occurring in areas of recent volcanism have characteristics indicating a volcanic relationship. However, the relationship is rarely so clear in regard to low and intermediate temperature thermal springs in areas without known volcanism. Conceivably the heat, but none of the mineral content of some of these springs, may be volcanic. In others, there may be a slight volcanic contribution of water and mineral content, but so diluted with meteoric water that it is difficult to recognize. What criteria are of greatest general value in determining the origin of such a thermal spring?

The authors believe the thermal gradient within a spring system, as compared to that of the surrounding region, promises to be a helpful criterion. Little attention has previously been focused on this subject, and the data are meager. Even the small amount of available information is restricted to shallow depths in a very few areas. It now appears that thermal springs owing their heat to a normal temperature increase with depth should have subnormal gradients within the spring system, and that springs with close volcanic connections do have abnormally high temperature gradients. Springs remotely related to volcanism may have gradients within their systems that are no higher than so-called normal gradients.

A direct volcanic contribution of water and mineral content is most likely to be indicated by higher chlorine, boron, sulphur and CO_2 than can otherwise be explained on the basis of composition of the meteoric water and bed rocks of the region. In areas where saline deposits are known and temperatures of springs are low to moderate, composition of the water is of little value, although comparison of proportions of individual constituents may be suggestive. Other criteria, such as composition of the gases, are probably of little value in moderate-temperature springs where there may be little or no volcanic contribution of water and mineral content. It is clear that in each case the problem is difficult and can be solved only by utilizing all of the available evidence.

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RESISTANCE TO FLOW IN ROUGH CHANNELS

Ralph W. Powell

Abstract--A formula, $C = -42 \log_{10} (C/\underline{R} + \epsilon/R)$, is developed for the resistance to flow in open channels, which it is believed is an improvement over Manning's or other formulas hitherto proposed. Graphs are given for the solution of this equation with three different groups of given quantities.

Introduction--The distinction between smooth and rough channel flow has already been drawn in a previous paper [POWELL, 1949]. Rough channel flow is sometimes called flow with fully developed turbulence. KEULEGAN [1938] gave the following formula for rough channel flow

$$V/V_f = 6.25 + 5.75 \log_{10} (R/k) \ldots\ldots (1)$$

where V is the mean forward velocity in the cross-section; V_f, the friction velocity = $\sqrt{\tau_0/\rho}$ = $\sqrt{RSg} = \sqrt{g}\, V/C$; C, the Chezy coefficient; R, the hydraulic radius; and k, the roughness measured as the diameter of the equivalent Nikuradse sand particle. Substituting $V_f = \sqrt{g}\, V/C$, this reduces to

$$C = 35.4 + 32.6 \log_{10} (R/k) \ldots\ldots (2)$$

But these formulas are based entirely on experiments with pipes, and on the assumption that the resistance to flow in a channel is the same as that in a pipe with the same R and k. In other words the effect of the free surface and of the angles between the walls and the bottom of the channel and between the side walls and the free surface, has been neglected. Experiments [POWELL, 1946] and computations of other experiments, show that this effect is quite considerable.

It seems necessary therefore to go back to Keulegan's more fundamental formula

$$V/V_f = a_r - b + b \log_e (R/k) + b\beta_k - \bar{\epsilon}\, V/V_f \ldots\ldots (3)$$

where a_r is an unknown which he later found to be a function of k, b is 2.50, the reciprocal of von Karman's universal constant, β_k is a function of the shape of the cross-section, and $\bar{\epsilon}$ measures the effect of the free surface and the corners. This can be rewritten in the form

$$C (1 + \bar{\epsilon}) = J + 2.5 \sqrt{g}\beta_k + 2.5 \sqrt{g}\ 2.3026 \log_{10} (R/k) \ldots\ldots (4)$$

where $J = \sqrt{g}\,(a_r - 2.5)$. If we assume that $\bar{\epsilon}$ has the same value as found for smooth channels, namely -0.208 (and the experiments discussed below confirm this), we have

$$C = 1.263\, J + 17.9\beta_k + 41.2 \log_{10} (R/k) \ldots\ldots (5)$$

Now let us change the scale by which the roughness is measured by making $\log_{10} k = \log_{10} \epsilon + 1.263\, J/41.2$. This simplifies (5) to

$$C = 17.9\beta_k + 41.2 \log_{10} (R/\epsilon) \ldots\ldots (6)$$

where ϵ represents the roughness to a somewhat different scale than Nikuradse's sand size. This is very similar to the formula

$$C = 40 \log_{10} (R/\epsilon) + 14.18\beta_k \ldots\ldots (7)$$

previously found [POWELL, 1946], but is probably somewhat more accurate.

Experimental results--In the study of smooth channels it was found that the shape correction $(17.9\beta_k)$ was hardly enough to be worth making, and that it could be omitted if the 41.2 coefficient were changed to 42.0. Making the same change here would give the equation

$$C = 42 \log_{10} (R/\epsilon) \ldots\ldots (8)$$

Eq. (6) and (8) were tested on the experimental data already published [POWELL, 1946]. The results are given in Table 1. Only four of the roughnesses were figured by (6), but since for these the fit was not as good as for (8), and since (8) is much simpler, it was thought unnecessary to compute the others. The average of the discrepancies of the values of C computed by (8) and the observed values, was 1.90 as compared with 2.14 as found by (7). This is not a significant improvement, especially as it is partly caused by using the best value of ϵ for each roughness, instead of forcing the value for II to be just twice the value for VIII (and similarly for I and IX and for X and XI) as was done before. But even with no increase in accuracy, (8) would be preferred over the previous formula because of its greater simplicity.

Table 1--Values of ϵ and discrepancies in C

Roughness	Value of ϵ by Eq (6)	By Eq (8)	
		Value of ϵ	Average discrepancy in C
	ft	ft	
I	0.00629 ± .00071[a]	0.00571 ± .00022[a]	0.64
II	. . . .	0.01029 ± .00096	1.72
III	. . . .	0.01415 ± .00112	1.59
IV	. . . .	0.01597 ± .00194	1.74
V	. . . .	0.01358 ± .00097	1.49
VI	. . . .	0.01133 ± .00100	1.58
VII	. . . .	0.00876 ± .00125	2.92
VIII	. . . .	0.00555 ± .00052	1.67
IX	0.00350 ± .00055	0.00312 ± .00046	2.71
X	0.00176 ± .00027	0.00163 ± .00024	2.60
XI	0.00339 ± .00043	0.00311 ± .00047	2.73

[a] The figures following the plus and minus signs are the average discrepancy from the mean.

The values given by JOHNSON [1944] for roughness D of his experiments were calculated by (8) and gave an ϵ of 0.00987 ± 0.00077 and an average discrepancy of the calculated C from the observed C of 1.44. It should be noted that Johnson found a value of k for this roughness of 0.0500 ft, so that from this one instance we have $k = 5.07\,\epsilon$. Many more experiments would be required to establish an accurate relationship between the two scales of roughness, and it is probable that the relationship is not linear. Of course the scale of roughness used in this paper is slightly different from that used before [POWELL, 1946].

One other set of data was also computed by (8), namely that given by LANSFORD and MITCHELL [1949] for cross-sections 2 and 4 of their flume. Cross-section 2 gave $\epsilon = 0.0130 \pm 0.0021$ with an average discrepancy in the computed values of C of 0.98. The best fit by Manning's formula was with n = 0.0193 and this gave an average discrepancy in the computed C's of 1.68. Cross-section 4 gave $\epsilon = 0.0485 \pm 0.0052$ with an average discrepancy in the computed C's of 1.04. The best fit by Manning's formula was with n = 0.0275 which gave an average discrepancy in the computed C's of 1.87. The question of comparison with Manning's formula will be treated more fully below. It should be noted that it is not necessary to know ϵ very accurately in order to get fairly accurate values of C (and hence of V and Q). Differentiating (8) with R kept constant gives

$$dC = -18.24\, d\epsilon/\epsilon \qquad (9)$$

so that a ten per cent change in ϵ would make a change in C of only 1.8 (in the opposite direction), and a five per cent change in ϵ would change C only 0.9.

Transition zone--In pipe flow it has been found that there is generally a transition zone between the smooth pipe and rough pipe zones, where the resistance depends on both the Reynolds' number and the relative roughness. The first satisfactory treatment of this was by COLBROOK [1939]. By analogy a similar formula for channels would be

$$C = -42 \log_{10} (C/\underline{\mathbf{R}} + \epsilon/R) \qquad (10)$$

where $\underline{\mathbf{R}}$, the Reynolds number, is taken as $4\,RV/\nu$. When $\epsilon = 0$, this gives the formula for smooth channels [POWELL, 1949], and when $\underline{\mathbf{R}} = \infty$ it reduces to (8). But as a matter of fact if we define the critical value of $\underline{\mathbf{R}}$ for any given value of R/ϵ to mean the $\underline{\mathbf{R}}$ which would give the same C by the smooth channel formula as by the rough channel formula, we may say that the $C/\underline{\mathbf{R}}$ term will be

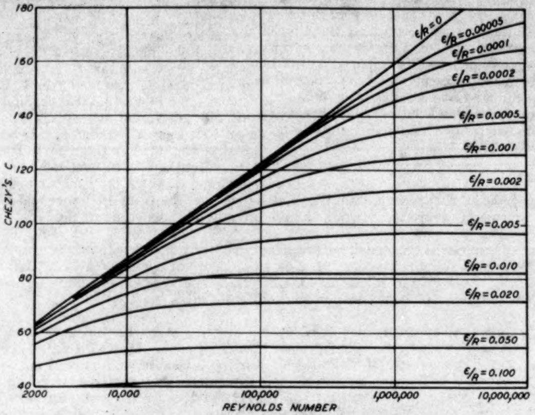

Fig. 1--Variation of C with relative roughness and Reynolds number, with Colbrook's transition

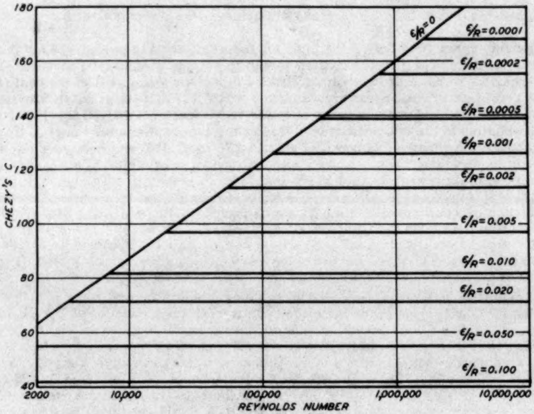

Fig. 2--Variation of C with relative roughness and Reynolds number, without transition

negligible for any value of R more than ten times the critical R, and that the ε/R term will be negligible for any value of R less than 1/10 the critical R.

But HARRIS [1949] has shown that in pipes where the roughness is quite uniformly distributed lengthwise along the pipe, there is practically no transition zone, but the flow changes abruptly from smooth to rough. The same thing is probably true in channels. At least in the five smoothest channels experimented on by the writer (I, VIII, IX, X, and XI,) where the C/R term was relatively larger, it was found that (10) gave a poorer fit than (8). The average discrepancy of the computed

Table 2--Tentative values of ϵ

Surface	ϵ
	ft
Neat cement surfaces	0.0002
Unplaned plank flumes	0.0010
Concrete-lined channels	0.004
Earth, straight and uniform	0.04
Dredged earth channels	0.10

C from the observed C was 2.12 for (10) and 1.97 for (8). It seemed therefore unnecessary to compute the other roughnesses by (10). But it should be emphasized that this was probably because this was an artificial roughness, duplicated accurately over and over again along the channel, and that in natural channels, and probably most artificial channels, (10) should be used. At least it is more conservative in that it gives lower flows in the transition zone. Figure 1 is a plotting of (10), and can be used to solve problems in which R, V, and ϵ are known and the slope is required. Figure 2 is a similar plotting without the transition zone and is for use only with artificial channels where the roughness is uniformly distributed along the length of the channel.

Much investigation will be needed before a satisfactory table of values of ϵ can be given, but Table 2 suggests some tentative values. The data on the River Beas given by THOMAS [1949] give an ϵ by (8) varying from 0.06 ft in July to 0.27 ft in December. Since these are at high Reynolds numbers, the results by (10) would have been practically the same.

Manning's formula--On December 4, 1889, at a meeting of the Institution of Civil Engineers of Ireland, a past president of the organization presented a paper [MANNING, 1891] containing several formulas for the velocity of flow in open channels. One, hidden away at the bottom of page 175, was $V = C\,S^{1/2}R^{2/3}$. He says he derived this formula independently in 1885, but admits that CUNNINGHAM [1882] had reported that HAGEN [1881] had deduced the same formula by least squares from Kutter's data. However this may be, it was soon found that when expressed in metric units, the C in this formula could be taken as the reciprocal of Kutter's n. Reduced to English units this became

$$V = 1.486\,R^{2/3}S^{1/2}/n \qquad (11)$$

which has for many years been called the Manning formula, and has become by far the most used of all channel formulas. The general opinion seems to be that although the theoretical basis of Manning's formula is weak, it serves very well as a close approximation of the best formulas available. (A recent exception to this view is taken by HARRIS [1949, p. 561]). Certainly one reason for the high opinion of Manning's formula is the treatment by KEULEGAN [1938] in the last section of his article. In the writer's opinion this is the least satisfactory part of this excellent paper. It starts with the numerical error of using 1.476 for 1.486, and then uses our (1) which we have pointed out is very inaccurate. Then it takes an average value (8.12) for a number which is admitted to vary at least from 7.78 to 8.36.

The equivalent of Keulegan's treatment results from equating the value of C from (11) to that from (8) and solving for n. The result may be written as

$$n = \phi\,\epsilon^{1/6} \qquad (12)$$

where $\phi = 0.03538\,(R/\epsilon)^{1/6}/\log_{10}(R/\epsilon)$. The value of ϕ is a minimum when R/ϵ is 403, and the minimum value is 0.0369. It is more for either smaller or larger values of R/ϵ, being 0.0411 for R/ϵ = 10,000, and 0.0519 for R/ϵ = 10. A plot of this variation is given in Figure 3. A much more useful graph is given in Figure 4, which shows the relationship of Manning's n to ϵ for various values of R. This shows that Manning's formula does not give even approximately the same results as the logarithmic formula. For ϵ = 0.020, the equivalent n is anything from 0.0192 up, depending on the value of R. Of course for the lower values of R, it might very well be that the Reynolds number would be small enough so that (8) would not apply. In this case it might happen that Manning's formula would give fairly good results, but there is no guarantee that it might not give impossibly high values of the velocity. For instance, with n = 0.010, R = 0.20 ft, and S = 0.00005, Manning's formula gives V = 0.359 ft/sec. But this is higher than the velocity could be, according to (10), even if the channel were perfectly smooth. For with ϵ = 0, (10) gives

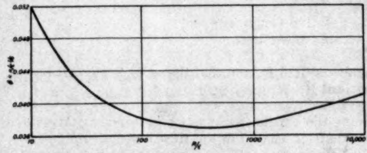

Fig. 3--Relationship of $\phi = n/\epsilon^{1/6}$ to R/ϵ

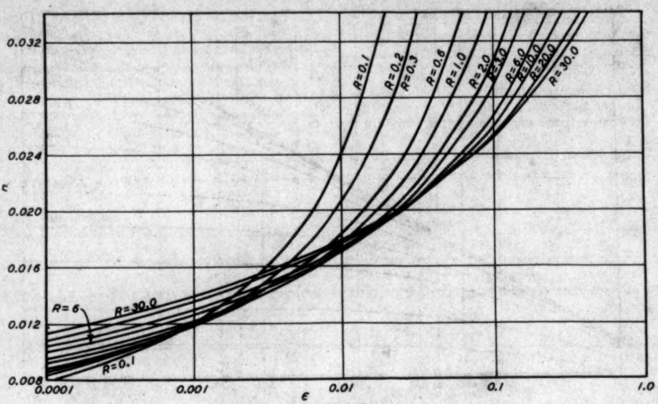

Fig. 4--Relation between Manning's n and ϵ in $C = 42 \log_{10} R/\epsilon$

(assuming $\nu = 0.0000125$) C = 96.9 and V = 0.306 ft/sec. With any appreciable roughness it would be less. For example if $\epsilon = 0.0004$ ft, which is the value that would be equivalent to n = 0.010 by (8), (10) gives C = 90.7, and V = 0.287 ft/sec. If this is correct, Manning's formula gives 25 per cent too large a velocity for this case.

Another comparison of the two formulas is to compare the rate of change of C with respect to R. Differentiating (8) with respect to R with ϵ fixed, gives

$$dC/dR = 18.24/R \tag{13}$$

Differentiating $C = 1.486\ R^{1/6}/n$ with respect to R with n fixed, gives

$$dC/dR = 0.248/n\ R^{5/6} \tag{14}$$

These two values agree when

$$R^{1/6} = 73.6\ n \tag{15}$$

For n = 0.010, this requires R to be 0.159 ft; for n = 0.015, R = 1.81 ft; for n = 0.020, R = 10.2 ft. When solved simultaneously with (12), this gives $R/\epsilon = 403$ as before.

It would seem that a real test of Manning's formula would be experiments in a large triangular flume of uniform roughness, where runs could be made at depths enough different so that at least a ten-fold variation in values of R could be obtained. If, then, C plotted against R on log-log paper gave a straight line with a slope of one sixth, Manning's formula would be vindicated. If, however, C plotted against R on semi-logarithmic paper gave a straight line with a rise of 42 between R = 0.1 and R = 1, (8) would be vindicated.

Graphs for practical use--When the slope is the unknown, Figure 1 may be used to give a direct solution. But when the velocity is unknown, the problem could be solved with Figure 1 only by a cut-and-try process. But this problem may be solved directly by means of Figure 5, which is a plotting of C against $\underline{R}/C$, as given by (10) for various values of ϵ/R. Since $C = V/(RS)^{0.5}$ and $\underline{R} = 4\ VR/\nu$

$$\underline{R}/C = 4\ R^{1.5}\ S^{0.5}/\nu \tag{16}$$

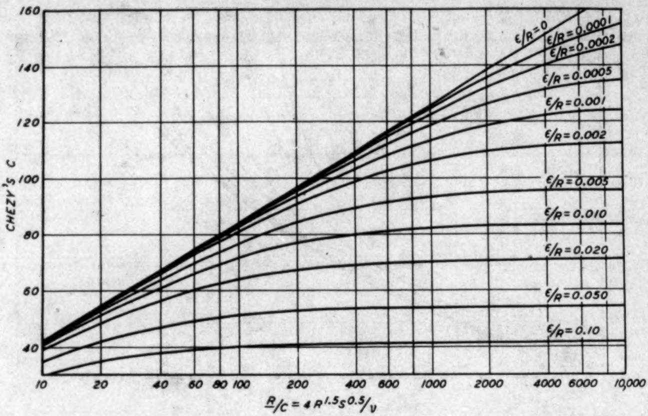

Fig. 5--Variation of C with roughness, hydraulic radius, and slope

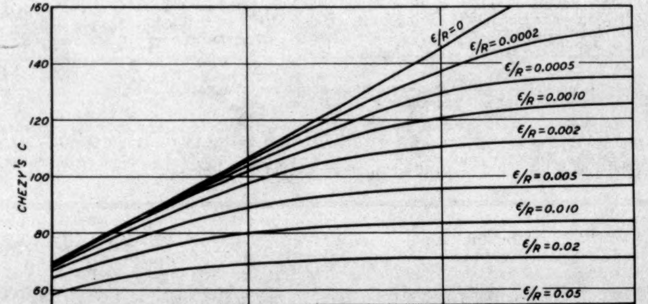

Fig. 6--Variation of C with quantity, slope, and roughness, for rectangular channel with depth half the width

so that if R, S, and ν are known, C may be found from Figure 5, and from it, V computed by the Chezy formula.

When R is the unknown, a direct solution can be found, as was pointed out by JOHNSON [1934], only by using the dimensionless number

$$\mathrm{I} = (Q^3\,S\,g)^{0.2}/\nu \quad \ldots\ldots (17)$$

There is of course no single solution unless the shape of the cross-section is specified. For example suppose that the section is to be a rectangle with the depth half the width. Then for a depth y, $A = 2\,y^2$, $P_w = 4\,y$, $R = y/2$, $Q = 2\,y^2\,V$ and

$$\underline{T} = 1.743\,\underline{R}/C^{0.4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

Figure 6 is a plotting of this function. For this particular section

$$R^{2.5} = Q/(8\,C\,S^{0.5}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

so that by using Figure 6 to get C, R can be obtained for any assumed ϵ/R, and then R determined more accurately by successive approximation. It must be emphasized that (18) and (19) and Figure 6 apply only to a rectangular section with the width twice the depth. But similar equations and graphs could be developed for other sections.

Illustrative example--Find the required size of a rectangular unplaned plank flume to carry 100 cfs with a slope of 0.00010, taking the kinematic viscosity of water as 0.0000125 sq ft/sec.

By (17), $\underline{T} = (100^3 \times 0.00010 \times 32.16)^{0.2}/0.0000125 = 404{,}000$. Assuming the width twice the depth and $\epsilon/R = 0.0005$, Figure 6 gives $C = 134.4$. Then by (19), $R^{2.5} = 100/8 \times 134.4 \times 0.01 = 9.30$ and $R = 2.44$ ft. By Table 2, $\epsilon = 0.001$ ft and $\epsilon/R = 0.00041$. Using Figure 6 and interpolating by eye, this gives $C = 139$. Then $R^{2.5} = 1250/139 = 8.99$, $R = 2.40$, $\epsilon/R = 0.00416$, and Figure 6 still gives $C = 139$. Therefore the channel should be 4.8 ft deep and 9.6 ft wide.

Conclusion--The paper on smooth channels considered the additional cases of non-turbulent and ultra-rapid flow. In the case of rough channels there is no occasion to consider non-turbulent flow, as with decreasing Reynolds numbers the flow changes to smooth channel flow before it becomes non-turbulent. The formulas given in this paper are for turbulent flow and apply both to tranquil and rapid (shooting) flow. Of the writer's 200 runs, 33 were rapid and the average discrepancy in the computed C's for these 33 runs was only 1.30, so that (8) fitted the rapid runs somewhat better than the tranquil ones.

It is probable that the resistance for ultra-rapid flow follows a somewhat different law, but unfortunately the writer has no data for this region.

In conclusion, it seems to the writer that (10) is a satisfactory formula for the resistance to turbulent flow in rough channels (although the constant 42 might be modified by more extensive and careful experiments); but that the list of proper values of ϵ to use in this formula needs further investigation.

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A GRAPHICAL METHOD FOR INVESTIGATING THE STABILITY OF FLOW IN OPEN CHANNELS OR IN CLOSED CONDUITS FLOWING PARTLY FULL

Francis F. Escoffier

Abstract--By making use of rectangular coordinates in which a certain stage variable ω is plotted as the horizontal coordinate and the mean velocity v as the vertical coordinate, some interesting properties of flowing water can be investigated. The method is then applied to a circular conduit flowing partly full.

The dynamic equation for flow in a uniform open channel is here used in the form

$$\partial V/\partial t + \alpha V(\partial V/\partial x) + (1 - \alpha)(Vw/A)\partial y/\partial t + g\, \partial y/\partial x = gi\,[1 - (V/V_n)^2] \quad \text{. . . . . (1)}$$

and the equation of continuity in the form

$$\partial A/\partial t + \partial Q/\partial x = I \quad \text{. . . . . (2)}$$

where
A = cross-sectional area of channel
g = acceleration of gravity
i = slope of channel bed
I = lateral inflow per unit length of channel
Q = discharge
t = time
V = mean velocity in channel cross section
V_n = normal velocity
w = width of free watersurface
x = distance along channel measured in downstream direction
y = stage variable
α = Boussinesq coefficient of velocity distribution

These two equations are substantially those given by KEULEGAN and PATTERSON [1943] except that the lateral inflow term I has been added to the equations of continuity. This term is useful in some problems where a large number of small tributaries enter a reach or where groundwater inflow is a factor.

If we rewrite (2) in the form

$$w\, \partial y/\partial t + A\, \partial V/\partial x + Vw\, \partial y/\partial x = I$$

multiply it by c/A and add the result to (1) we get

$$\partial V/\partial t + (\alpha V + c)\, \partial V/\partial x + [(1 - \alpha)\, V + c](w/A)\, \partial y/\partial t + (gA/w + cV)(w/A)\, \partial y/\partial x$$
$$= gi\,[1 - (V/V_n)^2] + cI/A \quad \text{. (3)}$$

The condition for the left-hand side of (3) to be a directional derivative is

$$gA/w + cV = (\alpha V + c)\,[(1 - \alpha)\, V + c] \quad \text{. . . . .(4)}$$

from which we get

$$c = \pm\sqrt{Ag/w + \alpha(\alpha - 1)V^2} \quad \text{. (5)}$$

which is the expression for the velocity of propagation and is also given by Keulegan and Patterson.

We now let

$$c' = \sqrt{Ag/w} \quad \text{. . . . . (6)}$$

and introduce a new stage variable ω defined by

$$\omega = \int_0^y c'(w/A)\,dy \tag{7}$$

By substituting these into (3) we obtain

$$\partial V/\partial t + (\alpha V \pm nc')\,\partial V/\partial x + (m \pm n)\,[\partial\omega/\partial t + (\alpha V \pm nc')\,\partial\omega/\partial x] = J \pm nL \tag{8}$$

where

$$m = (1 - \alpha)V/c' \tag{9}$$

$$n = \sqrt{1 + \alpha(\alpha - 1)(V/c')^2} \tag{10}$$

$$J = gi\,[1 - (V/V_n)^2] \tag{11}$$

and

$$L = c'I/A \tag{12}$$

Variables similar to the stage variable ω have been developed by Riemann and are mentioned by LAMB [1932] and BATEMAN [1932].

From (8) we derive the two characteristic equations

$$dt = dx/(\alpha V + nc') = [dV + (m + n)\,d\omega]/(J + nL) \tag{13}$$

and

$$dt = dx/(\alpha V - nc') = [dV + (m - n)\,d\omega]/(J - nL) \tag{14}$$

The characteristic defined by (13) which represents an impulse traveling downstream with a celerity $\alpha V + nc'$ shall be designated the forward characteristic. The one defined by (14) which represents an impulse traveling either upstream or downstream depending on whether αV is less or greater than nc', shall be designated the backward characteristic.

Since little is known concerning the value of α or the manner in which it may vary it will be here assumed as a simplification that α, m, and n are constants. In this connection it should be pointed out that the usual practice is to assume $\alpha = n = 1$ and $m = 0$.

In Figure 1 is shown in the (x, t) plane the effect of an initial disturbance extending from A to B at the moment $t = 0$ in an otherwise uniformly-flowing channel. It is assumed that no shock waves occur. The forward characteristic BD and the backward characteristic AC represent the initial impulses in the channel. The point P is taken a short distance upstream from BD and the point P′ a short distance downstream from AC. By making use of (13) and (14) we obtain

$$V_P + (m + n)\omega_P = V_M + (m + n)\omega_M + \int_M^P (J + nL)\,dt \tag{15}$$

$$V_P + (m - n)\omega_P = V_o + (m - n)\omega_o + \int_o^P (J - nL)\,dt \tag{16}$$

where V_o and ω_o are the values of V and ω in the undisturbed flow. Solving for $V_P - V_o$ and $\omega_P - \omega_o$ by means of these equations we get

$$V_P - V_o = [(n - m)(V_M - V_o) + (n + m)(n - m)(\omega_M - \omega_o) + (n - m)r + (n + m)s]/2n \tag{17}$$

$$\omega_P - \omega_o = [V_M - V_o + (n + m)(\omega_M - \omega_o) + r - s]/2n \tag{18}$$

where

$$r = \int_M^P (J + nL)\,dt \tag{19}$$

and

$$s = \int_{o}^{P} (J - nL)dt \qquad (20)$$

The corresponding solution for the point P′ is

$$V_{P'} - V_o = [(n + m)(V_{M'} - V_o) - (n + m)(n - m)(\omega_{M'} - \omega_o) + (n + m)r' + (n - m)s']/2n \qquad (21)$$

and

$$\omega_{P'} - \omega_o = -[V_{M'} - V_o - (n - m)(\omega_{M'} - \omega_o) + r' - s']/2n \qquad (22)$$

where

$$r' = \int_{M'}^{P'} (J - nL)dt \qquad (23)$$

and

$$s' = \int_{o'}^{P'} (J = nL)dt \qquad (24)$$

Eq. (17), (18), (21), and (22) admit of interesting application in rectangular coordinates where the plotted variables are ω and V. If in Figure 1 the point P is taken a sufficiently short distance from the forward characteristic BD for the quantity s in (17) and (18) to be a negligible quantity, we obtain by dividing (17) by (18),

$$(V_P - V_o)/(\omega_P - \omega_o) = n - m \qquad (25)$$

which indicates that the segment OP in the (ω, V) plane has a slope of n - m (see Fig. 2). For $\alpha = 0$ this would be a slope of unity. The corresponding equation, derived from (21) and (22), for the point P′ is

$$(V_{P'} - V_o)/(\omega_{P'} - \omega_o) = n - m \qquad (26)$$

In Figure 2 the variable V_n is represented as a function of ω by the curve HQ. The quantity J has a value of zero for all points on this curve, has minus values for all points above it, and has plus values for all points below it. In the following discussion it shall be assumed that the local inflow term L is zero so that the sign of r shall always be identical with that of J. It follows from a consideration of (17) and (18) that if P lies above HQ as shown in Figure 2, the values of $V_P - V_o$ and $\omega_P - \omega_o$ will diminish steadily with the progress of time. In other words, the effect of channel friction is to restore the quantities V and ω to their initial or undisturbed values V_o and ω_o.

This tendency of V and ω to return to their initial values can similarly be shown to be true for the point P′ and to remain true for both P and P′ when they fall below HQ. However, in Figure 3 where the line HQ has been drawn with a slope at the point 0 greater than n - m the situation as regard to P is changed. The sign of J and r has become plus and the quantities $V_P - V_o$ and

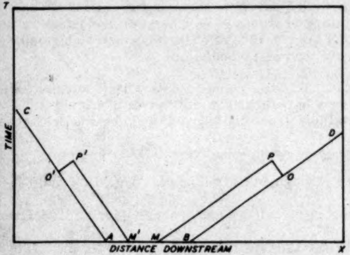

Fig. 1--Characteristics in the x-t plane

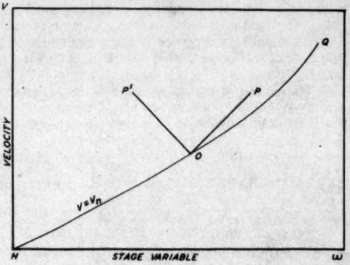

Fig. 2--Normal velocity curve

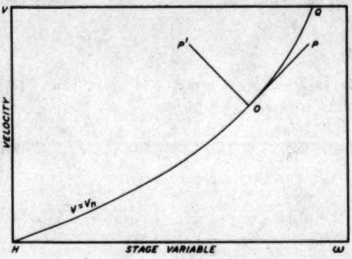

Fig. 3--Normal velocity curve for instability of the first kind

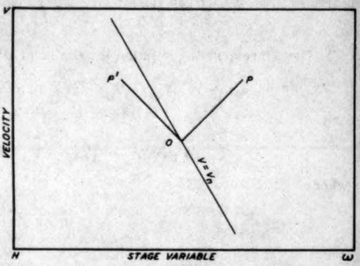

Fig. 4--Normal velocity curve for instability of the second kind

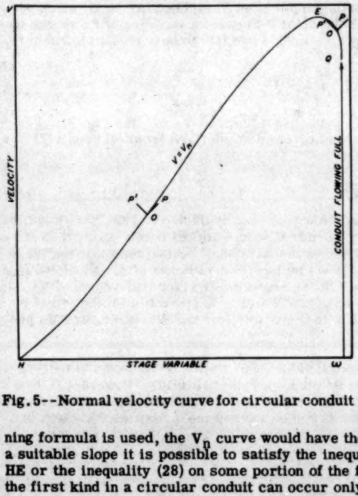

Fig. 5--Normal velocity curve for circular conduit

$\omega_P - \omega_O$ now tend to increase in value with the progress of time. In this case there is instability and any disturbance in the initially uniform flow will increase in magnitude as it progresses downstream.

The same is not true of the point P' and therefore the disturbance diminishes along its backward characteristic. Instability of this type, where

$$d\,V_n/d\omega > n - m \quad \ldots\ldots\ldots \quad (27)$$

shall be called instability of the first kind. A second type of instability, where

$$d\,V_n/d\omega < -(n + m) \quad \ldots\ldots \quad (28)$$

is indicated by (26) and shall here be called instability of the second kind. For this type of instability to exist it is necessary that some portion of the Vn curve have a minus slope as shown in Figure 4. This is possible in a closed conduit. If it is assumed that the normal velocity V_n varies as the two-thirds power of the hydraulic radius, as is assumed when the Manning formula is used, the V_n curve would have the form shown in Figure 5. By giving the conduit a suitable slope it is possible to satisfy the inequality (27) on some portion of the rising branch HE or the inequality (28) on some portion of the falling branch EQ. In other words, instability of the first kind in a circular conduit can occur only if the conduit is flowing less than eight-tenths full and, of the second kind only if it is flowing more than eight-tenths full.

The author knows of no case where waves of instability of the second kind have actually been observed. Since such waves would move quite slowly in comparison with waves of instability of the first kind they could develop to a greater extent in a short conduit and would be easier to observe.

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THE DYNAMIC CONCEPT OF SEDIMENT SOURCES

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Abstract--Using 200 upland sample areas and 23 valley cross-sections, a study was made of rates of sediment production and sediment sources from the headwaters of the Sangamon River of Illinois to its mouth. It was found that as the size of drainage area increases, the upland sources of sediment, such as sheet and gully erosion, become less important, while the bottomland sources, such as streambank erosion, floodplain scour, and valley trenching, become constantly more important. The writer maintains that this dynamic concept of changing sediment sources with changing size of drainage area applies not only to the Sangamon River Watershed, but to any watershed.

It was also found that the annual rate of sediment production in the Sangamon watershed decreases from an average of 2940 tons/sq mi of drainage area in the headwaters to 746 tons/sq mi at the mouth. This agrees with previous studies by the writer, which show that generally rates of sediment production decrease with increasing size of drainage area, for reasons which are set forth in the text.

The dynamic concept of sediment sources is compared with the previously widely used static concept based upon total erosion figures in a watershed, and the errors involved in the older concept exposed.

One of the more important conclusions drawn from this study is that streambank and other forms of bottomland erosion have not been given enough attention by the various agencies concerned with their control, in comparison with their importance when viewed in the light of the dynamic concept.

Introduction--In connection with controlling sediment damage of any kind, such as reservoir sedimentation, silting of drainage ditches, or infertile overwash deposits on bottomland, by watershed treatment, it is necessary to determine the sources of the sediment which is causing the damage. Only thus can a remedial program be planned which will treat these various sources in proportion to their importance. In the past, the relative importance of the various sediment sources has been generally determined simply by measuring or estimating the total erosion of various types, such as sheet, gully, and streambank erosion, which has taken place to date, and comparing these figures.

This paper is an attempt to show, using the drainage area of the Sangamon River in Illinois as an example, that sediment sources cannot be considered as static or constant as one proceeds downstream in a drainage system. Rather, they are constantly in process of change progressing downstream, with the upland sources, such as sheet erosion, gullies, and roadside erosion, diminishing in importance, and the bottomland sources, such as streambank erosion, floodplain scour, and valley trenching, increasing in importance. It should be pointed out that although in the Sangamon River Watershed an unusually high porportion of sediment is derived from sheet erosion and not so much from gully erosion, still the relationship between sediment derived from upland and bottomland sources is not far from average.

The basic data used in this study were obtained in June, 1948, in the Sangamon River Watershed during the course of the U. S. Department of Agriculture survey for "waterflow retardation and soil erosion prevention" under authority of the Flood Control Act of 1936. In this survey, detailed measurements in random samples were made on upland and bottomland to determine the amounts and rates of erosion and of deposition of various types. A study was also made of the land damages accompanying this erosion and deposition, such as land destruction by gullying and streambank erosion, and land damage by infertile overwash deposits, floodplain scour, and swamping (impairment of drainage by sediment deposits). The damage phase of this survey, however, was not used in this paper.

Method of study--Five major types of erosion were recognized and measured in this study: sheet erosion, gully erosion (including erosion of highway cuts and ditches), streambank erosion, floodplain scour, and valley trenching. The first two of these occur on the upland, and the others on bottomland. It is believed that these five types are generally sufficiently well understood so that redefinition here is unnecessary, except possibly for valley trenching, which is simply the development of channels on bottomland where none existed previously, or the deepening of existing channels. In the Sangamon Watershed only the first type of trenching, or the development of channels where none existed previously, was found.

The erosion or deposition measured in the field was the total accelerated or modern erosion or deposition which has occurred from the time the area was first settled, around 1818, up to the present time. The accelerated rate of erosion and sediment production in the Upper Mississippi drainage basin has been estimated [BRUNE, 1948] to be approximately 75 times the "geologic norm."

The term geologic norm has been discussed by **LOWDERMILK** and Others [1947]. The geologic norm of erosion and sediment production as used here refers to the annual rate of erosion and sediment production in the period immediately preceding the settling and clearing of this area by white men.

In measuring sheet and gully erosion and colluvial deposition on the uplands, 200 random sample areas of 1/4 square mile each were mapped in detail by soil scientists. Boring into the soil with augers, these men were able to determine, by textural, structural, and color differences, the depth of soil which had been removed by sheet erosion or deposited as upland colluvium. The dimensions of gullies were also determined and shown on the aerial photos, which had a scale of eight inches equals one mile. Historical records and the testimony of old residents showed that essentially all of the gully development in this watershed is modern; that is, it has all taken place since the area was settled by white men.

In measuring streambank erosion, floodplain scour, valley trenching, and alluvial deposition on the bottomlands, the mileage of stream valleys in each of 23 drainage area size classifications was determined. A random system of sample ranges, or valley cross-sections, was then laid out, so that each drainage area size classification was represented by one range. In other words, a synthetic stream was constructed, with sample ranges from the extreme headwaters to the mouth of the Sangamon River. The location of these ranges is shown in Figure 1.

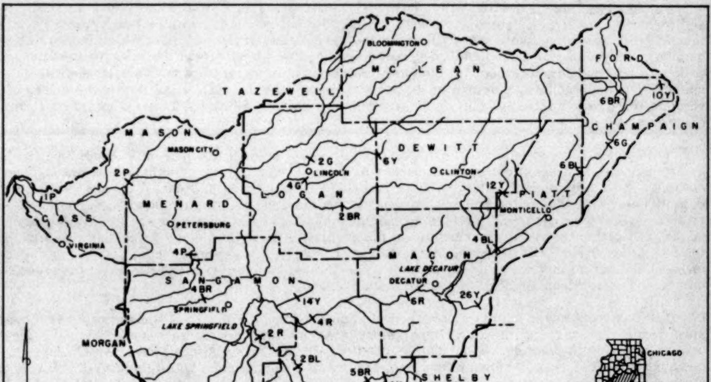
BLOOMINGTON
McLEAN
FORD
TAZEWELL
6BR
10Y
CHAMPAIGN
6G
MASON
MASON CITY
2G
LINCOLN
DEWITT
6Y
CLINTON
6BL
2P
4G
12Y
PIATT
1P
MENARD
LOGAN
MONTICELLO
CASS
PETERSBURG
2BR
VIRGINIA
4BL
4P
MACON
LAKE DECATUR
SANGAMON
DECATUR
4BR
6R
26Y
SPRINGFIELD
14Y
4R
LAKE SPRINGFIELD
2R
MORGAN
2BL
5BR
24Y
SHELBY
TAYLORVILLE
CHRISTIAN
22Y
MACOUPIN
MONTGOMERY
CHICAGO
SPRINGFIELD
ST LOUIS
LOCATION IN ILLINOIS

Fig. 1--Sangamon River Watershed location map, showing ranges used

The cross-sectional area of material removed by streambank erosion on each range was measured by comparing the amount of material removed with the amount which had been replaced on the other side of the stream, during the entire period since the area was settled, and also including any widening of the channel that may have taken place. In estimating the extent of channel widening that had occurred, certain reaches of stream which had been confined by trees and which had never suffered appreciable bank erosion were used for comparison.

Floodplain scour was found in the form of localized scour channels on the floodplains, sheet removal of large areas of soil from the bottomland, and erosion of low terrace soils which are subject to flooding. After gaining familiarity with the characteristics of the soil profiles involved, it was possible to estimate with considerable accuracy the depth of scour that had occurred on a given range. Old trees with exposed roots also served as excellent indicators of the depth of scour.

Valley trenches are simply gullies on bottomland. It was determined from historical records that the small amount of valley trenching found in the Sangamon Watershed has occurred since the area was settled. It was relatively simple to estimate the cross-sectional area of these modern trenches where they crossed sample ranges.

The measurement of modern sediment deposits was not difficult. In geologic times, before this area was settled, deposition of sediment upon bottomlands was very slow, with the result that dark grey or black, heavy-textured soils developed over most of the bottomland. When accelerated erosion and sedimentation began, brown, lighter-textured sediments began to be deposited upon these dark old soils. Usually a very sharp demarkation could be found at the contact between the old buried soil and the modern sediment. Buried fenceposts and modern artifacts also assisted in determining the depth of modern sediment at a number of points along each range.

The information gathered on bottomland erosion and deposition on each range was then expanded to totals, in acre feet, for each drainage area size classification, using the mileage of stream valley to which each range applied. The data from the upland sample areas were also expanded to totals.

The figures for total modern erosion and deposition of each type were then reduced to an annual basis. Historical records show that the watershed was first settled in 1818, or 131 years ago. However, accelerated erosion and sedimentation did not begin in earnest until some time later. For all types of erosion and deposition except gully erosion, the average period during which the erosion has occurred was computed to be 78 yr. In computing this figure, census data were used to plot "Improved land in farms" against "Years" from 1850 to the present. The area enclosed under the curve was assumed to have the same relationship to the area of improved land in farms in 1949 as the total erosion to date has to the erosion occurring this year. This method is based upon the assumption that use exposes land to erosion and that the magnitude of erosion (volume of sediment) will be in direct proportion to the area of land used or in farms. For gully erosion, a period of 50 yr was used in converting total volume of modern erosion to an annual rate.

Next, these data were reduced to a per-square-mile-of-watershed basis. Finally, they were converted from acre feet to tons, using an estimated specific weight of 90 lb/cu ft, based upon numerous measurements which have been made in this area.

Deposition of sediment in the two important reservoirs in the watershed, Lakes Decatur and Springfield, was also taken into account. The most recent sedimentation survey of Lake Decatur [BROWN, STALL, and DE TURK, 1947] showed that during the 24.2-yr period from 1922 to 1946 the average annual accumulation of sediment in the lake, per square mile of drainage area, was 237 tons. A detailed sedimentation survey of Lake Springfield in 1948 showed that during the 14.6-yr period from 1933 to 1948 the lake suffered an annual sediment accumulation of 71 tons/sq mi of drainage area.

Then, starting with the annual rate of erosion per square mile found on the upland for sheet and gully erosion, the study was carried downstream, range by range, through the synthetic stream system, to the mouth of the Sangamon River (drainage area 5410 sq mi). At each range, representing a certain number of miles of stream valley in a certain drainage area size classification, the amount of deposition, as colluvium, alluvium, or reservoir deposits, was noted. Then the amount of sediment from each source as found on the preceding (upstream) range was reduced in proportion as the total sediment load was reduced by deposition. The amounts of sediment added to the load by the bottomland sources, streambank erosion, floodplain scour, and valley trenching, were then added to the cumulative totals for sediment sources at the range.

Discussion and conclusions--In Table 1 the results of this study are summarized. The first thing that will be noticed, probably, is that the total annual rate of sediment production per square mile of drainage area decreases almost constantly from an average of 2940 tons on the upland slopes to 746 tons near the mouth of the Sangamon River. This general phenomenon of decreasing rates of sediment production with increasing size of drainage area has been proviously discussed in detail [BRUNE, 1949]. It is due to a combination of factors, including (1) decreasing stream gradients and consequent lower eroding and carrying capacity, and (2) decreasing frequency of basin-wide, high intensity storms, resulting in deposition of sediment in colluvial areas, on floodplains, and in channels.

Table 1--Relative sources of sediment, showing rate of production in tons per square mile of drainage area annualy

Range	Drainage area	Source of sediment					
		Sheet erosion	Gullies	Streambanks	Floodplain scour	Valley trenches	Total
Upland erosion surveys	sq mi						
	<1/4	2823	117	..	..	..	2940
10 Y	4	1310	55	..	..	42	1407
26 Y	8	1170	49	40	5	38	1302
6 Y	15	1170	49	67	5	38	1329
12 Y	18	1170	49	94	5	38	1356
14 Y	35	1110	47	89	5	36	1287
22 Y	88	1100	47	88	5	36	1276
24 Y	95	1090	47	87	5	36	1265
4 BR	110	1070	46	86	5	35	1242
5 BR	125	1060	46	85	5	35	1231
6 BR	160	1058	46	85	5	35	1229
6 G	325	1020	44	84	5	34	1187
2 G	330	994	43	90	7	33	1167
2 BR	430	985	43	92	8	33	1161
6 BL	510	963	42	90	8	32	1135
2 BL	600	960	42	91	9	32	1134
4 BL	680	955	42	91	10	32	1130
2 R	750	948	42	90	11	32	1123
4 G	930	938	41	96	13	31	1119
6 R	1050	895	39	92	13	30	1069
4 R	1250	834	36	112	23	28	1033
4 P	2960	788	34	106	22	26	976
2 P	5150	702	30	112	20	24	888
1 P	5350	590	25	94	17	20	746

In this case the sharp drop in total sediment load between 1/4 square mile and four square miles drainage area is caused by deposition of colluvium in small upland areas and deep alluvial deposits in small tributary headwaters. The rapid decline in sediment load downstream from range 4 P (2960-sq mi drainage area) is caused by the flattening river gradient, absence of any clearly defined channel downstream from range 1 P, and heavily timbered, swampy bottomland in the lower reaches of the Sangamon River, resulting in considerable deposition of sediment on the floodplain.

It will be noted in Table 1 that the rate of sediment production of that part of the sediment load attributable to the bottomland sources (streambank erosion, floodplain scour, and valley trenching) may rise or fall as the size of drainage area is increased, depending upon whether deposition or erosion is predominant in the particular drainage area size classification involved. On the other hand, the rate of sediment production of that part of the sediment load derived from upland sources (sheet and gully erosion) can only decrease, or at best remain constant, with increasing size of drainage area. This is true because when the drainage area increases to about one square mile, bottomland development begins to take place. From this point downstream, there is no further opportunity for upland erosion to take place, since the stream is flowing constantly in bottomland.

There is, of course, the possibility of small temporary increases in the rate of production of that part of the sediment load derived from upland sources where small tributaries join the main stream.

The reason that no decrease was found in the amount of sediment from sheet and gully erosion between eight and 18 sq mi is that the ranges representing these drainage area sizes were located in gorges or narrow bottomlands where conditions are unfavorable for deposition. At the same time, streambank erosion was found to be active in this reach, resulting in a temporary increase in the sediment load per square mile of drainage area.

Valley trenching, after its initial spurt at four square miles drainage area, declines steadily as a source of sediment. This is due to the fact that valley trenching was found only in small headwater drainage areas.

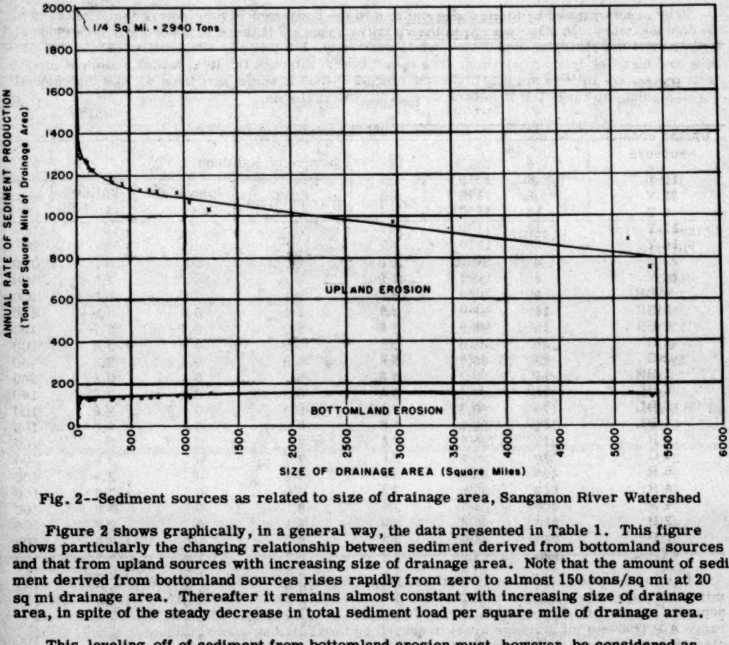

Fig. 2--Sediment sources as related to size of drainage area, Sangamon River Watershed

Figure 2 shows graphically, in a general way, the data presented in Table 1. This figure shows particularly the changing relationship between sediment derived from bottomland sources and that from upland sources with increasing size of drainage area. Note that the amount of sediment derived from bottomland sources rises rapidly from zero to almost 150 tons/sq mi at 20 sq mi drainage area. Thereafter it remains almost constant with increasing size of drainage area, in spite of the steady decrease in total sediment load per square mile of drainage area.

This leveling off of sediment from bottomland erosion must, however, be considered as strictly a coincidence. It takes place in the Sangamon Watershed, but in a watershed in which bottomland erosion is very active, such as the Soldier River of western Iowa where valley trenching is severe, and where there is no appreciable deposition on the floodplain, the amount of sediment derived from bottomland sources could be expected to increase constantly with increasing size of drainage area. On the other hand, in a watershed in which bottomland erosion is slight, sediment derived from bottomland sources could be expected to decline with increasing size of drainage area, although not to as marked a degree as sediment derived from upland sources.

The Sangamon River is not strictly typical of a normal river; because in 1905-1910 the portion from Mile 7 to Mile 33 (including ranges 1 P and 2 P) was straightened. Up to the time that the field work for this study was done (June 1948) the river was never straightened below Mile 7, although this has recently been done. The result was that floodwater passed rapidly through the

straightened stretch, with considerable channel erosion and widening. Below the straightened portion, however, the channel soon became obliterated by sediment deposits, and this entire area became a swamp. Eventually the effects of the channel sedimentation below the straightened portion backed up into the lower part of the straightened channel, causing it to lose much of its effectiveness for carrying water because of sediment deposits.

These abnormalities in the Sangamon River are reflected in Table 1. Range 2 P, which is located in the upper part of the straightened section, shows a temporary increase in annual sediment production per square mile from streambank erosion, but a net reduction in total annual sediment load per square mile. Range 1 P, located at the lower end of the straightened channel, shows a decided reduction in total annual sediment load per square mile of drainage area, with no erosion of any kind present.

The errors caused by these abnormalities in the Sangamon River, however, are believed to be compensatory. In other words, in the vicinity of range 2 P there is abnormally high channel erosion and low floodplain deposition, while near range 1 P there is abnormally low channel erosion and high floodplain deposition. The result is that although the total annual sediment load in tons per square mile is slightly higher at range 2 P than it would have been without the channel straightening, at range 1 P it is back down to the normal rate.

Table 2--Relative sources of sediment in per cent

Range	Drainage area	Source of sediment					
		Sheet erosion	Gullies	Streambanks	Floodplain scour	Valley trenches	Total
Upland erosion surveys	sq mi						
	<1/4	96.0	4.0	..	..	..	100
10 Y	4	93.1	3.9	..	..	3.0	100
26 Y	8	89.8	3.8	3.1	0.4	2.9	100
6 Y	15	88.0	3.7	5.0	0.4	2.9	100
12 Y	18	86.3	3.6	6.9	0.4	2.8	100
14 Y	35	86.3	3.6	6.9	0.4	2.8	100
22 Y	88	86.3	3.6	6.9	0.4	2.8	100
24 Y	95	86.3	3.6	6.9	0.4	2.8	100
4 BR	110	86.3	3.6	6.9	0.4	2.8	100
5 BR	125	86.3	3.6	6.9	0.4	2.8	100
6 BR	160	86.3	3.6	6.9	0.4	2.8	100
6 G	325	86.1	3.6	7.1	0.4	2.8	100
2 G	330	85.3	3.6	7.7	0.6	2.8	100
2 BR	430	85.0	3.6	7.9	0.7	2.8	100
6 BL	510	85.0	3.6	7.9	0.7	2.8	100
2 BL	600	84.7	3.6	8.1	0.8	2.8	100
4 BL	680	84.6	3.6	8.1	0.9	2.8	100
2 R	750	84.6	3.6	8.0	1.0	2.8	100
4 G	930	83.8	3.6	8.6	1.2	2.8	100
6 R	1050	83.8	3.6	8.6	1.2	2.8	100
4 R	1250	80.7	3.5	10.9	2.2	2.7	100
4 P	2960	80.7	3.5	10.9	2.2	2.7	100
2 P	5150	79.1	3.4	12.6	2.2	2.7	100
1 P	5350	79.1	3.4	12.6	2.2	2.7	100

Table 2 is similar to Table 1, except that the sediment sources are shown in per cent of the total rather than as actual amounts in tons per square mile annually. It will be noted that the proportion of sediment derived from streambank erosion and floodplain scour increases downstream rapidly on a percentage basis.

Where the percentage values remain the same from one range to another, as between 18- and 160-sq mi drainage area, the reason is that bottomland erosion is virtually absent in these reaches of stream. Deposition, however, was found to continue in these reaches, resulting in a steady decrease in total sediment load (see Table 1).

In Figure 3 the rates of sediment production found in this study for various sizes of drainage

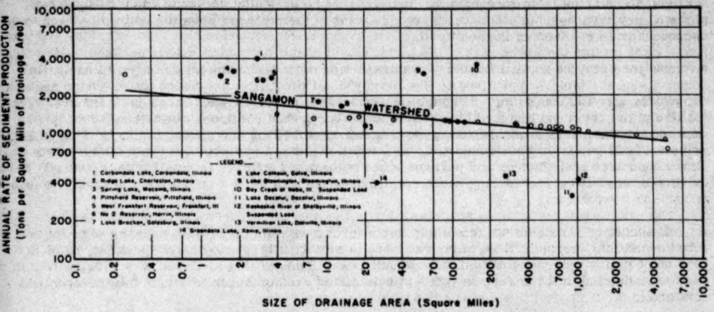

Fig. 3--Rates of sediment production in the Sangamon River Watershed compared with rates measured at other points in Illinois

area in the Sangamon River Watershed are compared with actual measured rates from 14 other points in Illinois. In 12 of these cases the data were taken from reservoir sedimentation surveys and in two cases from suspended load measurements.

This comparison affords a check on the reliability of the data computed for the Sangamon Watershed. It will be noted that the line for the Sangamon Watershed is about average when compared with records from the state as a whole. The phenomenon of decreasing rates of sediment production with increasing size of drainage area, previously mentioned, is also apparent in Figure 3.

If we were to consider only the total annual erosion of each type which takes place throughout the Sangamon River Watershed, without taking into account the constant change in source, which might be considered a reworking of the sediment which takes place as the size of drainage area increases, we would arrive at the following figures for sediment sources: sheet erosion, 91.6 per cent; gullies, 3.8 per cent; streambanks, 4.3 per cent; floodplain scour, 0.2 per cent; and valley trenches, 0.1 per cent.

Actually, by carrying our study through, range by range, from the headwaters to the mouth of the Sangamon River, we found that the percentage of sediment derived from sheet erosion, for example, varied from 96.0 on the uplands to 79.1 at the mouth of the river. At the same time the percentages of sediment derived from streambank erosion and floodplain scour were found to increase from zero to 12.6 and 2.2, respectively.

The total erosion figures given above actually do not even represent average sources of sediment for the watershed. They are closest to the values given in Table 2 for a drainage area size of approximately eight square miles, but even in this case they do not represent the correct proportion. In past watershed surveys for waterflow retardation and soil erosion prevention, sediment sources have been determined chiefly on the total erosion basis mentioned above. These figures have been considered average for the watershed as a whole and applied to all sediment damages within the watershed. The remedial programs recommended to alleviate these sediment damages have been based upon these average sources of sediment.

The result has been a minimizing of the importance of the bottomland sources of sediment in planning remedial programs for the relief of sediment damage. The important point to consider, of course, should be where the sediment damage is taking place; that is, what the average size of drainage area above the point of damage is. If the damage is occurring far downstream, the bottomland sources may become very important.

For example, consider the silting up of the Sangamon River straightened channel in the vicinity of range 2 P (drainage area 5150 sq mi). Table 2 indicates that the sources of the sediment which are causing this damage are approximately 79 per cent from sheet erosion, three per cent from gullies, 13 per cent from streambanks, two per cent from floodplain scour and three per cent from valley trenches. In other words, at this point in the drainage area the bottomland sources account for 18 per cent of the sediment.

Suppose that we go still farther downstream and consider sediment damages to navigation channels on the Mississippi River in the vicinity of Grafton, Illinois, where the drainage area is approximately 150,000 sq mi. Although this study does not cover such large drainage areas, it is likely that the trend found within the Sangamon Watershed continues downstream into larger drainage areas. It is probable that most of the sediment filling navigation channels on such large rivers is derived from bottomland sources. It follows that a remedial program which treats only the upland sediment sources and perhaps some bottomland sources in small tributaries will have a very limited effect in reducing sediment damages such as those to navigation channels, which occur far downstream.

It should be stressed that this study is concerned with the immediate sources of sediment, not the original sources. It is, of course, true that originally, geologically speaking, all of the sediment in any watershed came from upland areas. Some of this sediment was redeposited on the bottomlands, and then the various types of bottomland erosion began to attack this redeposited material.

It is equally true, however, that a program which controls sheet and gully erosion on the uplands will not thereby also control bank erosion, floodplain scour, and valley trenching on the bottomlands in the near future. The bottomland types of erosion will continue as long as there remains an acre of bottomland to be attacked. The bottomland which is now being eroded was, of course, derived from upland erosion occurring over thousands of years. Therefore, even if it were possible to stop completely sheet and gully erosion on the uplands, streambank and other types of bottomland erosion would continue, even at present accelerated rates, for many hundreds of years, until the bottomlands were largely destroyed for agricultural use. It is, therefore, necessary to treat not only the original sources of sediment on the upland, but also the immediate sources on the bottomland.

Acknowledgment--The writer desires to acknowledge particularly the work of G. Robert Hall, Soil Conservation Service, Milwaukee, Wisconsin, who made the measurements of various types of erosion and deposition on the sample bottomland ranges throughout the Sangamon Watershed.

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NUMERICAL SOLUTIONS FOR TILE DRAINAGE OF LAYERED SOILS

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(Journal Paper No. J-1730, Soils Subsection, Iowa Agricultural Experiment Station, Ames, Iowa, Project 998)

Abstract--The problem of the tile drainage of a uniform soil lying above an impermeable layer is solved by the relaxation method and compared with the analytical solution. The error in flow found by the relaxation method is less than four per cent, with greater accuracy obtainable if a finer net is used. The method is readily adaptable to complex problems involving layers of different permeabilities. As an example, the problem of the tile drainage of a two-layered soil with surface soil used for trench backfill, for which no analytical solution exists, is solved numerically.

Laplace's equation and the drainage problem--The solution of drainage problems usually calls for the determination of the amount of water that flows into a drainage utility, such as a tile drain, under certain specified conditions. Early investigators used the rather inaccurate Dupuit-Forchheimer assumptions to get approximate solutions. This method has been reviewed by RUSSELL [1934] and its defects pointed out by MUSKAT [1937]

SLICHTER [1899] pointed out that Laplace's equation applies to the flow of water in saturated soils, so that solutions of Laplace's equation, in two dimensions

$$\partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

will give flow lines, when ϕ represents a stream function; or if ϕ represents the hydraulic head, defined as the distance above a datum plane to which water will rise in an open pipe or piezometer, then the lines ϕ = constant are lines of equal hydraulic head. Slichter also pointed out that the flow of water in saturated soil is analogous to the flow of electricity in a conducting medium. This property has in fact been used by CHILDS [1943] in the study of a variety of drainage problems. The electric analogue requires equipment that may not be readily available; and its use for soils of non-uniform permeability requires considerable skill, especially if several permeabilities and irregular boundaries are involved.

Recently KIRKHAM [1949] has found solutions of certain drainage problems through analytical solutions of Laplace's equation. To simplify the mathematical treatment, such solutions are found only for soils of uniform permeability, and for configurations whose boundaries are more or less regular. Kirkham's solutions require much more advanced mathematics than the method to be presented.

To overcome the limitations of the above-mentioned methods, a numerical method, usually known as the "relaxation method," will be developed here and applied to a drainage problem. This numerical method is so simple that even a person with no mathematical training can work the problem once it is set up; and yet it is powerful enough to treat some of the most complex problems involving layers of different permeabilities with irregular boundary conditions, or with irregular interfaces between layers. Since the method is essentially one for solving Laplace's equation, it has many other applications to problems in temperature, electric potential and elasticity. SOUTHWELL [1945] and others have developed variations of this method which are more rapid and flexible or more general when used by trained computers, but the simplest form given here is probably best for those having infrequent uses for the method.

The relaxation method: a simple example--Let a square net or grid of uniformly spaced straight lines be drawn over a scale diagram of the region under consideration. In most problems the mesh or side of each square of the net can be chosen so that the boundary passes through corners of the squares. The procedure required in other cases (curved or oblique boundaries) will be given later. The net having been drawn, known values of the hydraulic head are entered at boundary points, and estimated values of the hydraulic head are assigned at each interior point of the net. In Figure 1 values of the hydraulic head at the eight points, A, B, . . . G, H, are taken as

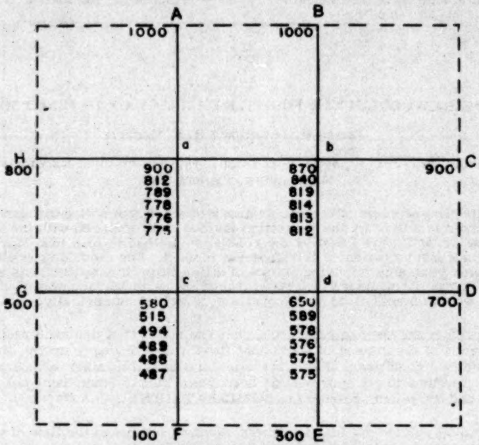
A
B
1000
1000
a
b
H
800
900
812
789
778
776
775
870
840
819
814
813
812
900
C
G
500
c
d
580
515
494
489
488
487
650
589
578
576
575
575
700
D
100
F
300
E

Fig. 1--Example of the relaxation method for solving Laplace's equation

known, and estimated values are entered at the top of the columns for interior points a, b, c, and d.

The interior points of the net are then traversed repeatedly, the value of the head at each point being replaced by the average of its four neighbors. Thus an improved value, ϕ'_a, for the hydraulic head at a is (for a development of this and other formulas of this paper see the appendix)

$$\phi'_a = (\phi_A + \phi_b + \phi_c + \phi_H)/4 \qquad (2)$$

or

$$\phi'_a = (1000 + 870 + 580 + 800)/4 = 812$$

Likewise, after ϕ'_a is found, an improved value at b is

$$\phi'_b = (1000 + 900 + 650 + 812)/4 = 840$$

and then

$$\phi'_c = (812 + 650 + 100 + 500)/4 = 515$$

Finally,

$$\phi'_d = (840 + 700 + 300 + 515)/4 = 589$$

Now we go back to point a and find a still better value

$$\phi''_a = (1000 + 840 + 515 + 800)/4 = 789$$

and continue in this way to improve values found for the hydraulic head at interior points. Values obtained after traversing the interior points five times change so little that the problem is regarded as solved, with precision limited by the coarseness of the net used. An attempt to improve the solution with this net would be useless, although with a finer net, the process might be continued for greater precision.

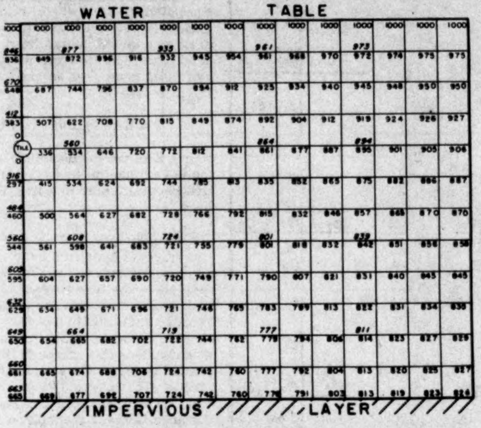
WATER TABLE
1000 1000 1000 1000 1000 1000 1000 1000 1000 1000 1000 1000 1000 1000 1000
846 877 935 961 973
836 849 872 896 916 932 945 954 961 968 970 972 974 975 975
670
648 687 744 796 837 870 894 912 925 934 940 945 948 950 950
412
383 507 622 708 770 815 849 874 892 904 912 919 924 926 927
0
560 864 894
TILE 336 534 646 720 772 812 841 861 877 887 895 901 905 906
0
316
297 415 534 624 692 744 785 813 835 852 865 875 882 886 887
464
460 500 564 627 682 728 766 792 815 832 846 857 865 870 870
560 608 724 801 839
544 561 598 641 683 721 755 779 801 818 832 842 851 856 858
605
595 604 627 657 690 720 749 771 790 807 821 831 840 845 845
632
629 634 649 671 696 721 746 765 783 799 813 822 831 834 835
649 664 719 777 811
650 654 665 682 702 722 744 762 779 794 806 814 823 827 829
660
661 665 674 688 706 724 742 760 777 792 804 813 820 825 827
663
665 669 677 692 707 724 742 760 778 791 803 813 819 823 824
IMPERVIOUS LAYER

Fig. 2--Tile drainage of a uniform ponded soil; tile depth, four feet; tile spacing, 28 ft; depth to impermeable layer, 12 ft; tile diameter, six inches

One boundary an impermeable layer--Consider next the tile drainage of a uniform soil with an impermeable layer at a depth of 12 ft, a tile spacing of 28 ft, and a tile depth of four feet (see Fig. 2). The surface of the water table is taken to be flat, and coincident with the soil surface. The tile line is assumed to have a hydraulic head of zero over its surface, while the head at the soil surface is 3.75 ft. For purposes of computation, the problem is solved as if the head at the soil surface were 1000 ft, so that all numbers in Figure 2 should be multiplied by 3.75/1000 to give the actual head.

No flow takes place across the face of the impermeable layer, so that the normal derivative of the hydraulic head along this line is zero. Likewise, no flow takes place across the lines of symmetry (flow lines in this case) passing vertically through the tile line, or passing vertically midway between two tile lines. Estimated values of the hydraulic head must be assigned along these three boundaries since the true values are not known. These estimated values are improved by taking the average of the sum of the values ot the two neighboring points along the boundary with twice the value at the interior neighboring point, or, referring to Figure 3b

$$\phi_o = (\phi_2 + \phi_4 + 2\phi_1)/4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Points in the interior of the region are adjusted as in (2), but points less than the distance a (see Fig. 3a) from the surface of the tile line must be treated by a special formula. In Figure 3c the improved value is given by

$$\phi_o = (\phi_1 + \phi_2 + \phi_3 + \phi_B/h)/(3 + 1/h) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

The solution is shown on Figure 2 by the vertical numerals below and to the left of net points. A comparison was made with the analytical solution given by **KIRKHAM** [1949], and his values for several points are shown by the slanted numerals above and to the left of the net points of Figure 2. The close agreement between these values and the numerically-obtained values is readily apparent. The greatest errors occur at points near the drain where the hydraulic head is changing rapidly. At points in the interior of the region the discrepancy is less than one per cent, while it goes up to seven per cent at points adjacent to the drain. It is possible to get more accurate numerical results in the region about the drain by reducing the net interval.

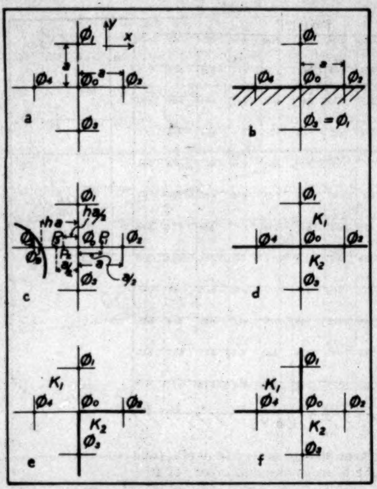

Fig. 3--Diagrams for derivations

Determination of the flow--While the equipotential plots are of interest, of far greater value is the fact that the flow into the drain can be calculated from the hydraulic-head plot. The flow across a line of equal hydraulic head over an area A is given by Darcy's law

$$Q = KA\,(\Delta\phi/\Delta n) \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (5)$$

where K is the permeability and $\Delta\phi/\Delta n$ is the rate of change of the hydraulic head in a direction perpendicular to a line of equal hydraulic head.

The surface of the water table lies on a line of equal hydraulic head and it was convenient to compute the flow across this surface since the flow across this surface will also be the flow into the drain tube. (A steady-state condition of flow is assumed, that is, the water table is assumed to remain at a constant level through the presence of ponded water.) The hydraulic head at the surface is everywhere 3.75 (the value 1000 in Fig. 2 was assumed for convenience); the distance between the top row of figures and the second row is $\Delta n = 1$ (foot); and the change of head $\Delta\phi$ is variable, ranging from 0.615 ft directly above the tile line to 0.094 ft halfway between the tile line.

If we are to find the flow across a strip of the surface one foot wide, then A equals one square foot, since the interval between the points along the surface is one foot. Taking K = 1 ft/day and adding up all of the Q's along the surface of the water table from a point over the tile to a point midway between the tiles, a value was obtained for one-half of the total flow into the drain tube. This result was then compared with the analytical result obtained by KIRKHAM [1949].

The result obtained by the numerical method was 6.840 cu ft/day while the analytical solution gave an answer of 6.644 cu ft/day. The error in the numerical answer was thus 3.0 per cent.

The two-layered soil with surface soil for trench backfill--There were 179 points at which the potential was determined in the problem of the preceding section and it took about 12 hr to get the answers. Values of the potential were calculated by means of Kirkham's equation at 26 points in about the same length of time. However, if a potential plot is not wanted, the total flow Q can be obtained in a few minutes from Kirkham's result.

On the other hand, solutions of problems involving soils with layers of various permeabilities are difficult or impossible to obtain analytically, especially if the interface between the two layers is irregular. The real power of the numerical method is realized in the solution of these more complex problems.

In southern Iowa, Missouri, and southern Illinois, there are large areas of soils that are difficult to drain because of dense heavy clay subsoil. To indicate how the numerical method is used in such cases, let us assume that tile drains are to be laid at 20-ft intervals in soil consisting of a 24-inch layer of permeability K_1, below which there is an 8.5-ft layer of permeability K_2, all underlaid by an impermeable layer. The six-inch tiles are laid on the bottom of a trench 18 inches wide and 48 inches deep, half imbedded in the subsoil. The material used to fill the trench is assumed to have permeability K_1.

This drainage problem was solved for the three permeability ratios $K_1/K_2 = 5, 10, 100$. For points interior to regions of uniform permeability, the procedure is exactly as we have used before, but special formulas are required to deal with points lying on the interface. (For convenience the net was so arranged that the interface did not separate two adjacent points.) Referring to Figure 3d,

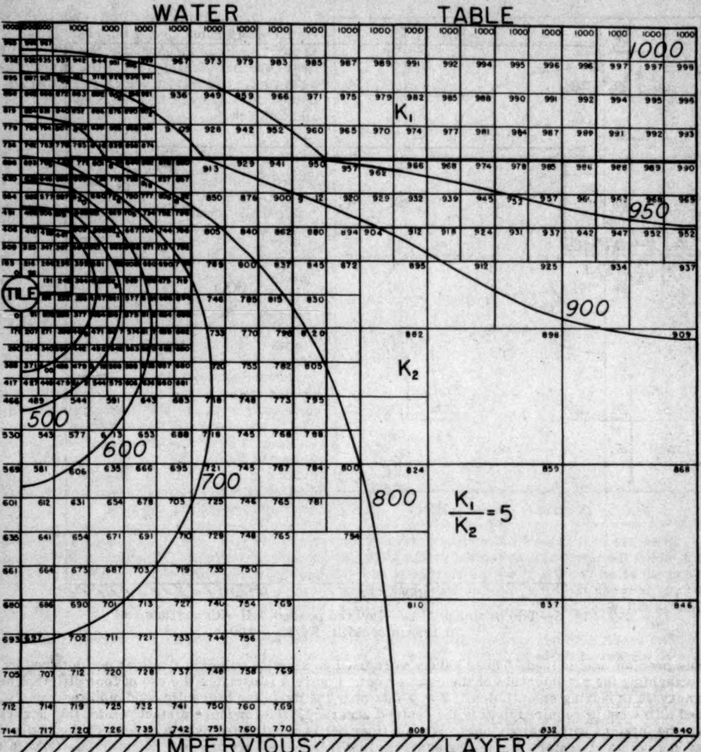
WATER TABLE
1000
950
900
K₁
K₂
800
700
600
500
TILE
K₁/K₂ = 5
IMPERVIOUS LAYER

Fig. 4--Tile drainage of two-layered ponded soil with surface soil for trench backfill; $K_1/K_2 = 5$

we have the formula

$$\phi_o = [(K_1 + K_2)(\phi_2 + \phi_4) + 2(K_1\phi_1 + K_2\phi_3)]/4(K_1 + K_2) \quad \text{. . . . . (6)}$$

and for Figures 3e and 3f

$$\phi_o = [2K_1(\phi_1 + \phi_4) + (K_1 + K_2)(\phi_2 + \phi_3)]/2(3K_1 + K_2) \quad \text{. . . . . (7)}$$

and

$$\phi_o = [2K_2(\phi_2 + \phi_3) + (K_1 + K_2)(\phi_1 + \phi_4)]/2(3K_2 + K_1) \quad \text{. . . . . (8)}$$

for upper and lower corner points, respectively.

A very coarse net was first laid down over the region, and, with the aid of (6), (7), and (8),

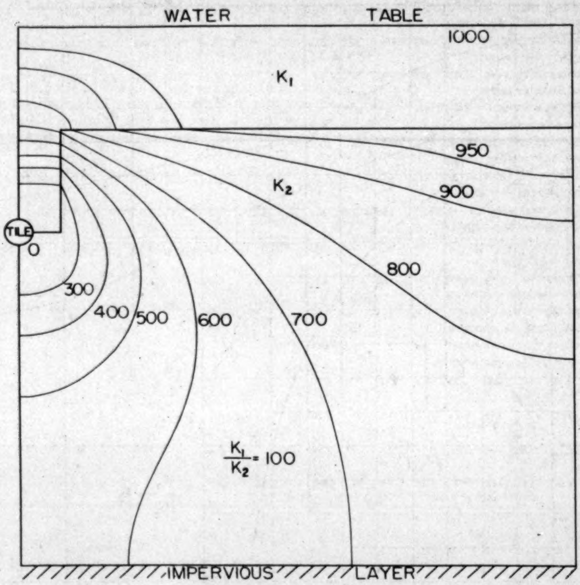

Fig. 5--Tile drainage of two-layered ponded soil with surface soil for trench backfill; $K_1/K_2 = 100$

this problem was solved. These values were used as starting values for a finer net obtained by subdividing the net intervals of the coarser net. Figure 4 illustrates the use of four different net intervals in solving one problem. For points near the drain the hydraulic head changes rapidly, and a fine net is required to give the desired accuracy. But for more distant points, this accuracy can be attained with coarser nets. When a finer net is constructed over a portion of the region by adding points at centers of existing squares, it will be necessary at some points to use (2) with the four diagonally adjacent points, because the values at vertically and horizontally adjacent points, which would normally be used, will not be known. Since, however, Laplace's equation is unchanged on rotation of axes, (2) is still valid. Figure 5 shows the lines of equal head for K_1/K_2 equal to 100. Table 1 gives the values of Q obtained in the several cases.

Table 1--Results of numerical analysis

K_1/K_2	Q	Relative flow
	cu ft/day	
1	6.289[a]	100.0
5	2.905	46.1
10	2.667	42.4
100	2.586	41.1

[a] Calculated analytically from the equation of KIRKHAM [1949].

Discussion--It is clear from the results in Table 1 that when the ratios of the permeabilities K_1/K_2 is five or greater, very little water flows through the subsoil. We may conclud therefore, that on a planosol which has a permeability ratio of at least ten, very little flow takes place through the subsoil when tile drains are placed (as in Fig. 4 and 5) at four-foot dep Most of the water must flow laterally through the surface soil in order to reach the more permeab trench-fill material.

To compare the relative advantages of placing the tile at the bottom of a four-foot trench, as was done in the problems here solved, or of placing the tile at a two-foot depth with half of the tile embedded in the impermeable layer, the drain flux in the latter was calculated by means of KIRKHAM'S [1947] equation, to be

$$Q = 2.371 \text{ cu ft/day}$$

Comparing this result with the numerically-obtained answer for the two-layered soil with permeability ratio $K_1/K_2 = 100$ (see Table 1), one finds that the flow into the tile is reduced by a factor $(2.586 - 2.371)/2.586$ or 11.1 per cent by placing the tile at a two-foot depth on the surface of an impermeable layer, instead of in the trench backfilled with surface soil, at the four-foot depth.

The small increase in flow obtained by placing the tile at a four-foot depth would probably not justify the additional cost of digging the deeper trench.

Appendix

For sufficiently small values of a (see Fig. 3a), the derivative of the function $\phi(x, y)$, with respect to x, is approximately

$$[\phi(x + a, y) - \phi(x, y)]/a$$

and the second derivative is approximately

$$\left\{[\phi(x + a, y) - \phi(x, y)]/a - [\phi(x, y) - \phi(x - a, y)]/a\right\}/a$$

so that from Figure 3a

$$\partial^2\phi/\partial x^2 \doteq (\phi_2 + \phi_4 - 2\phi_o)/a^2$$

$$\partial^2\phi/\partial y^2 \doteq (\phi_1 + \phi_3 - 2\phi_o)/a^2$$

and Laplace's equation (1) becomes, after reduction,

$$\phi_o = (\phi_1 + \phi_2 + \phi_3 + \phi_4)/4$$

This is the basic formula (2) for solving Laplace's equation by the relaxation method.

Special formulas are developed in the same way. If the points ϕ_o, ϕ_2, ϕ_4 lie on the boundary of an impermeable layer (see Fig. 3b), then a fictitious value of ϕ_1 is assigned to ϕ_3, and (2) reduces to (3). (The quantities ϕ_o, ϕ_1, are actually values of ϕ, but it is convenient to use them also to designate points of the net.) But if the boundary separates two points of the net, say ϕ_4 and ϕ_o as in Figure 3c, then

$$\partial\phi/\partial x]_{P_1} \doteq (\phi_2 - \phi_o)/a$$

$$\partial\phi/\partial x]_{P_2} \doteq (\phi_o - \phi_B)/ha$$

Now let us assume that $\partial\phi/\partial x]_{P_2} = \partial\phi/\partial x]_{P_3}$, that is, that the function ϕ varies linearly across the interval from the boundary to ϕ_o. In that case,

$$\partial^2\phi/\partial x^2 \doteq [(\phi_2 - \phi_o)/a - (\phi_o - \phi_B)/ha]/a$$

and Laplace's equation becomes

$$\phi_1 + \phi_2 + \phi_3 + \phi_B/h - (3 + 1/h)\,\phi_o = 0$$

After reduction, we have Eq. (4).

When materials of different permeability are present, the more general form of Laplace's equation

$$\partial(K\,\partial\phi/\partial x)/\partial x + \partial(K\,\partial\phi/\partial y)/\partial y = 0$$

with $K = K(x, y)$, must be used. Referring to Figure 3e, involving an upper corner point, we shall develop (7). The development of (6) and (8) is similar. Between ϕ_2 and ϕ_o the average permeability

$(K_1 + K_2)/2$ is used, so that

$$K\,\partial\phi/\partial x \doteq (K_1 + K_2)\,(\phi_2 - \phi_o)/2a$$

But between ϕ_o and ϕ_4, K is the constant, K_1. Thus

$$\partial(K\,\partial\phi/\partial x)/\partial x \doteq [(K_1 + K_2)\,(\phi_2 - \phi_o)/2a - K_1\,(\phi_o - \phi_4)/a]/a$$

With a similar formula,

$$\partial(K\,\partial\phi/\partial y)/\partial y \doteq [K_1\,(\phi_1 - \phi_o)/a - (K_1 + K_2)\,(\phi_o - \phi_3)/2a]/a$$

we find for Laplace's equation

$$2K_1\,(\phi_1 + \phi_4) + (K_1 + K_2)\,(\phi_2 + \phi_3) - (6K_1 + 2K_2)\,\phi_o = 0$$

which is equivalent to (7).

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FLUID RESISTANCE OF COMPOSITE ROUGHNESS

H. A. Einstein and R. B. Banks

Abstract--A series of flume experiments was conducted to determine the composite resistance of different types of roughness opposing the flow of water through an open channel. Each of the roughness types investigated possessed a distinctive geometrical shape and each type contributed resistance to the flow. The different types of roughness were acting on the same surface and each could be assumed to be uniformly distributed over the surface. The combined or composite resistance was sought in terms of the resistances which the individual roughness types exerted.

Within the range of the variables tested, it was found that the total resistance exerted by combined types of roughness is equal to the sum of the resistance forces exerted by each type individually.

Introduction--Previous investigations of the problem of artificial roughness in channels appears to be confined to the study of the effect of uniform roughness elements. Early experimental work by BAZIN [1865] and the basic research of NIKURADSE [1933], FROMM [1923], HOPF [1923], and others, established roughness criteria as did more recent investigations of roughness employing various types of uniform roughness elements. Among these studies are the works of SCHLICHTING [1937], experimenting with spheres, cones and angles in a closed conduit, FAGE [1933] employing small pyramids, HOUGHTON [1932] using a corrugated sheet and JOHNSON [1944] and POWELL [1946] experimenting with small rectangular sills.

The question of composite roughness, in which more than one type of resistance element is present, was suggested in a study of the Salinas River. In this river, vegetation and sand bars are present which naturally exert resistance to the flow. In this case the total force opposing the flow consists of (a) the resistance exerted by the particles composing the river bed and sides; (b) the geometrical or form resistance of the bars; and (c) the resistance exerted by the vegetation. It is natural to inquire whether the total friction can be expressed in some way as the sum of the individual resistances.

It is most significant to note that these different frictional devices are not arranged side by side such as the various grain sizes of a sediment mixture, but are actually superimposed and thus acting on the same area. Such a superposition may best be exemplified by a sheet of sand paper. If this sand paper is laid flat on a flat bottom it gives a roughness according to its grain. This same sand paper, on the other hand, may be arranged in the shape of a corrugated sheet. In this case the corrugation and the grain represent two different types of roughness which are superimposed over one another. Each type may be measured individually and combined; comparison of these results will determine the effects of these different frictional devices and how they must be described such that their effects may be easily combined.

In the present study, the following types of resistance were used (see Fig. 1): (a) blocks without offset and without pegs; (b) blocks without block offset, combined with various peg densities and patterns; (c) blocks with alternate blocks offset, without pegs; and (d) blocks with alternate blocks offset, combined with various peg densities and patterns.

It is evident that in types (a), (b), and (c) the individual resistance due to each type of roughness can be determined. Arrangement (a) allows the determination of the friction along the plane concrete surface. Comparison of arrangements (a) and (b) gives the influence of the pegs and comparison of (a) and (c) that of the offset. Arrangement (d) then permits the comparison of the sum of the individual resistances with their combined or composite resistance.

Theory--Since most of the experiments were conducted at near-uniform flow conditions, Eq (1) for non-uniform flow of O'BRIEN and HICKOX [1937, p. 274] was employed in the determination of the energy slope, S_e

$$S_e = S_w - \alpha Q^2 (S_w - S_b)/gb^2d^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

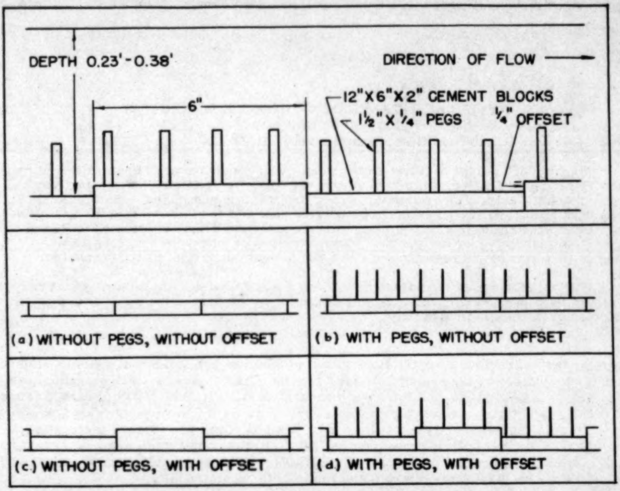
DEPTH 0.23'-0.38'
DIRECTION OF FLOW
6"
12"X6"X2" CEMENT BLOCKS
1½"X¼" PEGS
¼" OFFSET
(a) WITHOUT PEGS, WITHOUT OFFSET
(b) WITH PEGS, WITHOUT OFFSET
(c) WITHOUT PEGS, WITH OFFSET
(d) WITH PEGS, WITH OFFSET

Fig. 1--Types of roughness investigated

in which S_w is the water surface slope, $\alpha = \int v^3 dA/Av_m^3$, the velocity head correction; Q, the discharge, cfs; b, the flume width; d, the average depth of flow; and S_b, the slope of the channel bottom. A value of $\alpha = 1.10$ was obtained by graphical integration in preliminary runs and this value was used in all subsequent calculation.

In order to eliminate the effects due to the side walls, whose roughness was constant during the tests (Manning coefficient of wall, $n_w = 0.0087$), the method of side wall elimination of EINSTEIN [1942] was employed in calculating the Manning roughness coefficient of the bed, n_b, and the friction factors. These values were calculated from the expressions

$$n_b = 1.486\, R_b^{2/3}\, S_e^{1/2}/V \qquad (2)$$

$$f = 8R_b\, g\, S_e/V^2 \qquad (3)$$

R_b is the hydraulic radius of the bed and is equal to A_b/b, where A_b is the cross sectional area of the channel corresponding to the bed. Though the parameters n_b and f express the friction, it is more appropriate to consider the actual resistance exerted by the various roughness elements. In order to express this concept quantatively, the following assumptions were made: (a) The blocks offer only shear resistance along the bed surface, expressible in the form

$$T = T_o \qquad (4)$$

where T is the resistance measured in lbs/ft^2 exerted by the bed on which the bed shear T_o acts. (b) The offsets and the pegs cause only form resistance acting on the flow of the free stream and each can be expressed in the form

$$T = C_D\, \rho\, SV^2/2 \qquad (5)$$

in which C_D is a drag coefficient, ρ, the density, S, the area of the obstruction per unit bed area projected on a plane normal to the flow and V the average velocity. (c) The bed area occupied by the pegs is small compared to the total area. The reduction of actual cement bed area due to the presence of the pegs, therefore, may be neglected.

Identifying the various expressions with subscripts b, p, and o, to identify their reference to the block surface, pegs and offset, respectively, it follows that

$$T_b = T_o = wR_b\, S_e = \rho\, V^2 f_b/8 \tag{6a}$$

in which w is the unit weight of water and other terms as defined previously. Similar expressions can be made for T_{bo}, T_{bp}, and T_{bpo} which denote the resistances per square foot of bed area due to the blocks and offset, the blocks and pegs, and the blocks, pegs and offset, respectively. That is, the following hydraulic radii may be defined by

$$T_{bo} = wR_{bo}S_e \tag{6b}$$

$$T_{bp} = wR_{bp}S_e \tag{6c}$$

$$T_{bpo} = wR_{bpo}S_e \tag{6d}$$

In these equations, R, with the respective subscripts, represents the hydraulic radius of the bed due to the presence of the block surface, the offset and the pegs. Considering (6d) as the most general case it is seen that the hydraulic radius R_{bpo} is equal to the cross sectional area A_{bpo} corresponding to the bed divided by the wetted perimeter, that is,

$$R_{bpo} = A_{bpo}/b$$

The total cross sectional area of the channel is equal to A. Eliminating the side wall effects, the area corresponding the the bed is

$$A_{bpo} = A - A_w$$

This area, A_{bpo}, can be represented schematically consisting of areas, A_b, A_o, and A_p each corresponding to the appropriate roughness element, that is, to the block surface, the offset and the pegs, respectively. This is analogous to the assignment of the area A_w to the side wall influence. Then,

$$A_{bpo} = A_b + A_o + A_p \tag{7}$$

These areas are representative of the energy expended on the particular roughness element. The energy represented by each of these areas is nearly uniformly distributed across the channel and hence each area is assumed to possess the same wetted perimeter, b. Consequently, the hydraulic radii corresponding to the block surface, the offset and the pegs are respectively

$$R_b = A_b/b; \quad R_o = A_o/b; \quad R_p = A_p/b \tag{8}$$

As A_{bpo} has been defined as the sum of the areas pertaining to the blocks, offset, and pegs combined, (7), the sum of the three hydraulic radii of (8) equals

$$R_b + R_o + R_p = (A_b + A_o + A_p)/b = A_{bpo}/b = R_{bpo} \tag{9}$$

These hydraulic radii may be added directly only because they refer to the same wetted perimeter. This combination is basically different from that between bed and side wall friction.

From (9) and (6d) the force T_{bpo} may now be expressed as follows

$$T_{bpo} = wR_{bpo}\, S_e = wR_b\, S_e + wR_p\, S_e + wR_o\, S_e = T_b + T_p + T_o \tag{10}$$

or as the sum of the resistances which the block surface, pegs, and the offset exert individually. The assumption is made that these three individual resistance forces, T_b, T_p, and T_o are mutually independent and may be expressed by (6) and (10). This is verified experimentally in the following measurements.

Experimental procedure--The series of experiments were conducted in a 17-ft flume, 12 inches wide and 18 inches deep. The sides were of painted sheet metal. Water was recirculated through the channel from a constant head reservoir through a six-inch pipe to a stilling basin at the entrance of the flume. Transition guide vanes directed the flow from the basin into the channel and the flow was permitted free outfall at the exit. Discharge was determined by means of a calibrated orifice. The water-surface slope was obtained with a battery of seven point gage wells connected to tap holes in the side wall of the flume. The first connection was at a point 2.5 ft from the stilling basin. Thereafter, connections were spaced 1.75 ft apart with the last well connected to a point 3.25 ft from the outfall. Prior to the installation of the blocks in the channel, the Manning coefficient of the bottom and sides was determined for use in eliminating the side wall effects.

As stated in the introduction, the investigation of the composite roughness was divided into four series of experiments. In the first series, the channel bottom area was composed of 12 × 6 × 2-inch cement blocks (with the six-inch dimension parallel to the direction of flow) placed side by side throughout the channel reach. By means of a rack supporting these blocks, the bottom was adjusted to the desired slope which was kept constant throughout all the series. For each run the discharge was determined and the slope of the water surface obtained by least squares from the well point gage readings. The energy slope was computed with (1), using the average depth of flow. By eliminating the effects due to the side walls, the hydraulic radius of the bed was obtained and the Manning coefficient, friction factor and total resistance computed. The water temperature recorded for computation of Reynolds' number, varied from 72° to 75° F.

For the second series, pegs (1 1/2 inch height × 1/4 inch diameter) were screwed into holes on the block surface in various patterns and densities (0.25 to 10 pegs sq/ft). The experimental procedure was the same. In the third series, alternate blocks were offset vertically 1/4 inch with the pegs removed. In the fourth series, pegs were again placed on the blocks in patterns and densities similar to those of the second series.

To prevent flow beneath the blocks the edges between the side walls and blocks and between adjacent blocks were sealed. In the offset series, the joints between blocks were sealed and then trimmed to preserve the sharp corner offsets.

Experimental results--From the first series of experiments an average value of $f_b = 0.0208$ was obtained. With this value of f_b, (6a) becomes

$$T_b = \rho f_b V^2/8 = 0.00505\ V^2 \quad \ldots \quad (11)$$

Since,

$$T_{bp} = T_b + T_p = \rho f_b\ V^2/8 + C_{Dp}\ N \rho S_p\ V^2/2$$

where N is the peg density, in pegs per sq ft of bed area. Then,

$$\begin{aligned} T_{bp} &= [\ \rho f_b/8 + C_{Dp}\ \rho N\ S_p/2]\ V^2 \\ &= [0.00505 + C_{Dp}\ \rho N\ S_p/2]\ V^2 \quad \ldots \quad (12) \end{aligned}$$

From the second series the value of $C_{Dp}\ \rho\ S_p/2$ was determined by plotting T_{bp}/V^2 versus N and obtaining the line of best fit by the least squares method. This value, $C_{Dp}\ \rho\ S_p/2$, represents the slope of (12) and was computed to be 0.00175 from which $C_{Dp} = 0.652$. (Naturally, for a given shape this coefficient may vary with Reynolds' number, $\underline{R}$. However, since the experiments were all conducted at nearly equal $\underline{R}$, this is assumed constant ($\underline{R} = 300{,}000$). From a published curve of C_D versus $\underline{R}$ for flow around immersed cylinders, C_{Dp} has the approximate value of 0.7 [ROUSE, 1938, p. 217]). Then,

$$T_{bp} = [0.00505 + 0.00175\ N]\ V^2 \quad \ldots \quad (13)$$

Similarly, for the runs with alternate blocks offset, the magnitude of $\rho f_b/8 + C_{Do}\ \rho\ S_o/2$ was determined to be 0.0122. Then $C_{Do}\ \rho S_o/2 = 0.0122 - 0.00505$, $C_{Do}\ \rho S_o/2 = 0.00715$ and $C_{Do} = 0.362$. Then

$$\begin{aligned} T_{bpo} &= T_b + T_o + T_p \\ &= [\ \rho f_b/8 + C_{Do}\ \ \rho S_o/2 + C_{Dp}\ \ \rho N\ S_p/2] \quad \ldots \quad (14) \\ &= [0.0122 + 0.00175\ N]\ V^2 \quad \ldots \quad (15) \end{aligned}$$

Equations 13 and 15 are shown in Figure 2. Points shown in this figure represent data obtained experimentally. The lower points, computed from (6c), are for the series without the offset and the upper points, computed from (6d) are for the series with the offset. The lower line (Eq 13)

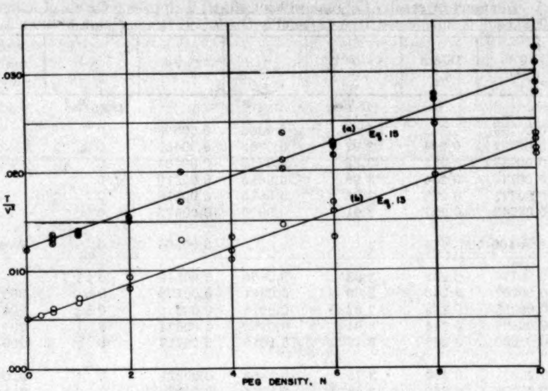

Fig. 2--Comparison of resistances for (a) with offset and (b) without offset for various peg densities

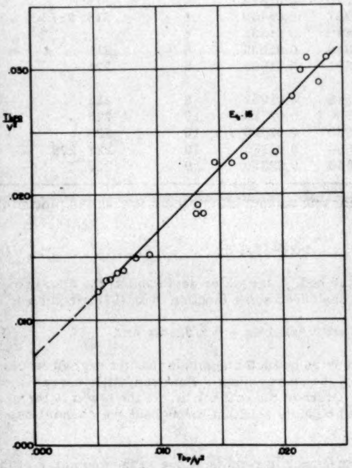

Fig. 3--Comparison of resistance with offset with resistance without offset for identical peg arrangements

was computed by least squares from the experimental data. The upper curve (Eq. 15) was drawn parallel to the lower curve at a distance $C_{Do}\rho S_o/2$ which represents the resistance offered by the offset. For completeness, experimental data are presented in Tables 1 and 2.

The foregoing analysis considered none of the geometrical characteristics of the arrangements of the pegs in the channel. In Figure 3 a comparison is made between the resistance exerted by a certain peg arrangement in which there was an offset with the resistance exerted by an identical pattern with no offset at equal flow velocities. If the value of T_{bpo}/V^2 for a certain peg arrangement is plotted as the ordinate and the value of T_{bp}/V^2 as the abscissa for the same arrangement, then it follows from (13) and (15) that

$$T_{bpo}/V^2 = 0.00715 + T_{bp}/V^2 \quad . . . . . . (16)$$

This equation and the points obtained from experimentation are shown in Figure 3.

There are several probable explanations for the scatter of the points in Figures 2 and 3. First, it was assumed that the velocity head correction factor, α, was a constant (α = 1.10) throughout the test reach. However, differentiating the total energy

Table 1--Summary of results for runs without offset. S_p (projected area of a single peg) = 0.00278 ft^2; S_o (projected area of the offset) = 0.0204 ft^2; b (flume width) = 1.007 ft

Run	S_e	Depth d_m	Velocity V	T	T/V^2	N	Corresponding run with offset[a] (see Table 2)
		ft	ft/sec	lb/ft^2		pegs/ft^2	
100	0.00765	0.235	3.98	0.0805	0.00508	0	....
101	0.00732	0.234	3.97	0.0768	0.00486	0	....
102	0.00737	0.233	3.98	0.0769	0.00484	0	....
103	0.00774	0.234	3.96	0.0815	0.00519	0	....
104	0.00773	0.233	3.98	0.0818	0.00516	0	....
105	0.00743	0.237	3.91	0.0800	0.00523	0	....
Average, runs 100-105			...	....	0.00506	0	Average 200-202
106	0.00774	0.238	3.93	0.0840	0.00546	0.25	....
107	0.00797	0.240	3.90	0.0881	0.00579	0.5	203
108	0.00815	0.239	3.87	0.0915	0.00610	0.5	204, 205
109	0.00858	0.244	3.84	0.0990	0.00674	1	207, 208
110	0.00860	0.246	3.77	0.1018	0.00717	1	209, 206
111	0.00941	0.250	3.75	0.1152	0.00821	2	210
112	0.00906	0.260	3.56	0.1174	0.00929	2	211, 212
113	0.00975	0.283	3.29	0.1421	0.01311	3	213, 214
114	0.00993	0.266	3.49	0.1342	0.01105	4	....
115	0.01019	0.270	3.45	0.1406	0.01181	4	....
116	0.00884	0.293	3.17	0.1329	0.01322	4	215
117	0.00796	0.313	2.97	0.1287	0.01460	5	216, 217
118	0.01024	0.278	3.33	0.1479	0.01331	6	....
119	0.00775	0.325	2.86	0.1310	0.01599	6	219
120	0.00803	0.327	2.85	0.1371	0.01695	6	221
121	0.00880	0.330	2.81	0.1542	0.01952	8	223
122	0.00845	0.352	2.64	0.1600	0.02210	10	229
123	0.00860	0.351	2.65	0.1620	0.02315	10	230
124	0.00833	0.348	2.68	0.1550	0.02165	10	227, 228
125	0.00862	0.353	2.63	0.1640	0.02370	10	229

[a] Where two corresponding runs with offset exist for one without offset, their average is plotted in Figure 3.

$$dE/dx = dz/dx + dy/dx + d(\alpha V^2/2g)/dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

in which E is the total energy, z, the elevation of the bed, y, the water depth and x, the distance along the flume, it is seen that if in general α is considered some function of x, (17) becomes

$$dE/dx = dz/dx + dy/dx + \alpha d(V^2/2g)/dx + (V^2/2g)\, d\alpha/dx \quad \ldots\ldots\ldots (18)$$

and an additional term is introduced. This term may be of such magnitude that its neglect would introduce a considerable error in the calculation of the energy slope. However, little correction can be made without the detailed knowledge of the nature of the α function. In the few α determinations made by the writers, however, it seemed to be quite constant throughout the channel especially at large peg density.

Undoubtedly the major source of apparent discrepancy in results is due to the geometry of the peg arrangements. It is evident that two different patterns of arrangement with the same density will exert different resistances. This accounts for variations in total resistance among points possessing the same peg density, N (see Fig. 2). This was confirmed by comparing the peg arrangements for the points situated above the curve with the arrangements for the points situated below.

Table 2--Summary of results for runs with offset

Run	S_e	Depth d_m	Velocity V	T	T/V^2	N
		ft	ft/sec	lb/ft^2		$pegs/ft^2$
200	0.00928	0.281	3.30	0.1338	0.01227	0
201	0.00937	0.279	3.32	0.1340	0.01214	0
202	0.00945	0.279	3.32	0.1345	0.01221	0
Average, runs 200-202			. . .	. . . .	0.01220	0
203	0.00916	0.288	3.22	0.1359	0.01306	0.5
204	0.00924	0.290	3.20	0.1385	0.01351	0.5
205	0.00924	0.285	3.25	0.1352	0.01276	0.5
206	0.00881	0.297	3.31	0.1351	0.01380	1
207	0.00880	0.295	3.14	0.1334	0.01350	1
208	0.00887	0.296	3.13	0.1357	0.01385	1
209	0.00874	0.299	3.10	0.1351	0.01404	1
210	0.00844	0.307	3.03	0.1349	0.01491	2
211	0.00830	0.313	2.97	0.1351	0.01539	2
212	0.00836	0.310	2.99	0.1345	0.01500	2
213	0.00810	0.341	2.72	0.1467	0.01990	3
214	0.00819	0.324	2.86	0.1393	0.01700	3
215	0.00814	0.336	2.76	0.1450	0.01904	4
216	0.00858	0.354	2.62	0.1640	0.02385	5
217	0.00835	0.344	2.70	0.1538	0.02110	5
218	0.00816	0.341	2.71	0.1437	0.02020	5
219	0.00848	0.348	2.66	0.1590	0.02245	6
220	0.00837	0.345	2.69	0.1547	0.02150	6
221	0.00847	0.352	2.64	0.1584	0.02290	6
222	0.00839	0.350	2.65	0.1578	0.02250	6
223	0.00844	0.353	2.63	0.1605	0.02330	8
224	0.00918	0.362	2.54	0.1818	0.02770	8
225	0.00845	0.359	2.58	0.1645	0.02470	8
226	0.00899	0.362	2.56	0.1776	0.02720	8
227	0.00947	0.375	2.47	0.1969	0.03230	10
228	0.00912	0.362	2.56	0.1803	0.02745	10
229	0.00934	0.371	2.49	0.1911	0.03080	10
230	0.00921	0.366	2.53	0.1845	0.02880	10

However, the problem of developing a true roughness parameter which includes both the geometry of peg arrangement and their density has not been successful. In Figure 3 in which the effect of the geometry has apparently been eliminated, the experimental points appear to be in better agreement with the computed relationship.

Conclusions--Evidently the conditions imposed by the assumptions stated above are of a limited nature. However, within the range of the parameters investigated there appears to be consistency in results. The conclusion that the total resistance can be expressed as the sum of the individual resistances is logical as long as the separate roughness elements do not exert any mutual interference on one another. Over quite a broad range of peg densities this independence of influence of roughness types exists; at least, they are effectively independent. For example, consider the two bed types: (1) without offset and without pegs; and (2) with offset and without pegs. In the latter the offset exerts an additional resistance due to the form resistance of the offset and separation at the down stream edge of the raised block. This separation zone reduces the surface effect in some places by diverting the flow from these areas but appears to concentrate the flow sufficiently in other areas to increase the shear so that the total shear force remains about constant. The same reasoning applies to the interference between the block surface shear and the peg resistance. Thus it appears logical to expect an additive property among the separate resistances until the types exhibit excessive mutual interference.

It is of interest to investigate the distribution of the total resistance among the various roughness elements and this is possible when the amount of resistance which each element contributes to the total is known. From (14) it is seen that the three terms of the right hand member express the block surface resistance, the offset resistance and the peg resistance, respectively. The ratio of each term to the total expression, then represents the per cent of total resistance which each type exerts. This distribution is a function of the peg density, N, and hence the effect of the introduction or removal of pegs on resistance distribution can be investigated. This concept may be of value in predicting the effect of the introduction of artificial roughness on the stabilization of a river bed.

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A THEORY OF FRACTURE OF MATERIALS AND ITS APPLICATION TO GEOLOGY

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Abstract--A theory of fracture of materials is developed which encompasses shear and tension failure. The probability of fracture occurring across a given surface element in an isotropic medium is assumed to be a general function of the limiting normal and tangential stresses across that element. A modification is later introduced to allow for anisotropy wherein the medium has a single plane of weakness.

The probable fracture at a point in an isotropic medium is predicted to take place across a pair of conjugate surface elements, equally inclined to the direction of the maximum pressure at an angle which is a function of the limiting stresses as well as constants of the medium. In anisotropic media failure is predicted generally across a single surface element. Applied to geology the theory accounts for certain types of faults, jointing in rocks, for certain effects of increase in depth, and for the deflection or termination of a fault by an earlier fault or shear zone.

Introduction--The study of the formation of faults, shears, and joints in rocks brings us back to fundamental questions concerning failure of materials, about which much has been done and written. Despite the difference in scale generally involved, the laws governing failure of large volumes of rock in terms of which the geologist thinks, and of brittle materials employed in engineering, are very similar. It might therefore be profitable to investigate the geologic consequences of the theories proposed in the latter realm on failure of brittle materials.

In this paper we shall follow a new mathematical approach to the problem, one that will lead us to the results of MOHR [1928] with an important generalizing condition such as will enable us to treat both shear and tension failure in the same theory.

Although much work has been done since Mohr's time by Hencky, von Kármán, Nadai, and others, on the failure of materials (see for example the volumes of Zeitschrift für angew. Math und Mechanik, 1921-1933), much still remains to be done before the theory will fit all the experimental evidence. A general survey of the results and shortcomings of these theories has been presented by NADAI [1933].

The recent theories of failure based on conceptions of plasticity and creep of materials strictly do not apply to the realm of normal or brittle rock failure such as occurs near surface.

Mohr's theorem--This paper really constitutes a generalized extension of Mohr's theorem.

In all states of stress which lead to rupture in isotropic material, the surface elements of rupture at every point in the medium are parallel to the intermediate principal stress and inclined at an angle $\phi < 45°$ to the direction of the greatest pressure, that is, smallest principal stress, if the positive sign is taken for tension. The angle ϕ is determined solely by constants of the material. Thus, rupture in an isotropic material under the action of a stress distribution is predicted to occur along a surface composed of such surface elements at each point of the medium. In a homogeneous isotropic medium with a homogeneous stress distribution the surfaces of rupture will then be planes. The main difficulty of this theory is that the magnitude of the intermediate stress does not enter at all. Moreover, in purely tensile fracture in brittle material it is well known that the surfaces of rupture are perpendicular to the direction of tension, which is contradictory to the theorem of Mohr.

We shall find, however, that very simple considerations will enable us to arrive at a theorem similar to that of Mohr with the exception that the angle ϕ becomes a function of the limiting stresses, removing this contradiction. The ensuing theory is, in its geologic application, a macroscopic one. That is, it is assumed to apply only statistically over regions of crystalline media that are large in proportion to the crystal size.

The first section of the work deals with a medium that is, at least statistically, homogeneous and isotropic, while the latter part treats a medium with a single plane of weakness. Geologically speaking this may be interpreted as rocks such as gneiss, slate, schists, and others whose origin or past history has given them a plane of weakness. From the first part one may deduce, as does ANDERSON [1942], using a more particular and very early theory due to Navier, the three classes of faults that may occur and their relation to the principal stresses causing them. The latter part enables us to offer explanations for such things as the termination of one fault by another, and the deflection of a fault by an earlier shear zone or fault.

A new theory of failure of isotropic media--For the necessary theoretical background the reader is referred to any one of the standard treatises on elasticity [SOKOLNIKOFF, 1946].

Let the principal stresses at a point P in the medium under consideration be σ_1, σ_2, σ_3 (positive sign for tension). The stress across any surface element with normal (n_1, n_2, n_3) relative to the principal axes of stress can be split up into a normal and a shear stress across the surface element. The magnitude of the former is

$$N = \sigma_1 n_1^2 + \sigma_2 n_2^2 + \sigma_3 n_3^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

and of the latter

$$T = [n_1^2 n_2^2 (\sigma_1 - \sigma_2)^2 + n_1^2 n_3^2 (\sigma_1 - \sigma_3)^2 + n_2^2 n_3^2 (\sigma_2 - \sigma_3)^2]^{1/2} \ldots\ldots \quad (2)$$

(always considered as positive).

We will try to predict the most probable surface elements of failure of the medium at P from a knowledge of the stresses acting and by making somewhat general assumptions about the limiting conditions necessary to cause failure.

It is a matter of experience that the probability of failure depends on several factors. In a given medium for a fixed shear stress, the probability increases with the magnitude of the normal stress across the surface element, and incresses with the magnitude of the shear stress, the normal stress remaining constant. In anisotropic media the probability of failure across a given surface element may also be an explicit function of the orientation of the element for N and T fixed

As our hypotheses about the failure of isotropic media we shall express the last paragraph in mathematical form.

Hypotheses--(1) There exists a well defined function, $F = F(N, T)$, that is, dependent explicitly only on N and T, (and hence implicitly on the orientation of the surface element) as well as on constants of the medium, which expresses the probability of fracture across the surface element. The surface element for which F first reaches the value unity will be called π_F, the "surface element of fracture." (2) The partial derivatives, $\partial F/\partial N = F_N$, $\partial F/\partial T = F_T$ are assumed to be positive.

On the point of failure F must be a maximum for π_F. Thus, in order to determine π_F we should try to find those values of (n_1, n_2, n_3) which maximize $F = F(N, T)$ subject to the auxiliary condition that

$$n_1^2 + n_2^2 + n_3^2 - 1 = 0$$

By Lagrange's method of undetermined multipliers [GOURSAT, 1904] the maximizing conditions become

$$n_1 \left\{ [n_2^2 (\sigma_1 - \sigma_2)^2 + n_3^2 (\sigma_1 - \sigma_3)^2] F_N/T + 2\sigma_1 F_N - 2\lambda \right\} = 0 \ldots\ldots\ldots\ldots \quad (3)$$

$$n_2 \left\{ [n_1^2 (\sigma_1 - \sigma_2)^2 + n_3^2 (\sigma_2 - \sigma_3)^2] F_T/T + 2\sigma_2 F_N - 2\lambda \right\} = 0 \ldots\ldots\ldots\ldots \quad (4)$$

$$n_3 \left\{ [n_1^2 (\sigma_1 - \sigma_3)^2 + n_2^2 (\sigma_2 - \sigma_3)^2] F_T/T + 2\sigma_3 F_N - 2\lambda \right\} = 0 \ldots\ldots\ldots\ldots \quad (5)$$

where λ is the undetermined multiplier. In a non-degenerate case ($\sigma_1 \neq \sigma_2 \neq \sigma_3$) these are only compatible if one of n_i vanishes. If we assume for the sake of definiteness $n_3 = 0$ and $\sigma_1 < \sigma_2$ then it follows

$$n_1^2 = 1/2 + F_N T/(\sigma_1 - \sigma_2) F_T \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

But, since T, F_T, F_N are all positive

$$n_1^2 < 1/2 \qquad (7)$$

Suppose we denote by the dimensionless number m the ratio F_N/F_T. We find after substituting $[n_1^2\, n_2^2\, (\sigma_1 - \sigma_2)^2]^{1/2}$ for T finally that

$$4n_1^4\,(1 + m^2) - 4n_1^2\,(1 + m^2) + 1 = 0$$

This gives us two values for n_1^2, but knowing that $n_1^2 < 1/2$ we can remove the extraneous root, getting

$$n_1 = \pm \left\{1/2 - 1/2\,[1 - 1/(1 + m^2)]^{1/2}\right\}^{1/2} \qquad (8)$$

Since m is a function of n_1, this is an implicit equation for n_1. Thus, corresponding to the solution n_1 of this equation there are two possible surfaces πF which we will call "conjugate surface elements of fracture," inclined at the same angle.

$$\phi = \cos^{-1}\,[1/2 + m/2\,(1 + m^2)^{1/2}]^{1/2}$$

to the direction of the greater principal pressure σ_1, and parallel to the direction of σ_3. We may rewrite this as

$$\tan 2\phi = 1/m \qquad (9)$$

Similarly, corresponding to the other possible choices of $n_1 = 0$ and $n_2 = 0$ we have two other possible sets of conjugate planes characterized by a relative maximum of F. To determine which of the three sets affords an absolute maximum of F we must assume for definiteness

$$\sigma_1 < \sigma_3 < \sigma_2.$$

When $n_3 = 0$ we have $N = (\sigma_1 - \sigma_2)\,n_1^2 + \sigma_2$ and $T^2 = (\sigma_1 - \sigma_2)^2\,n_1^2\,(1 - n_1^2)$ whereas in the parallel case of $n_2 = 0$ we would have

$$N' = (\sigma_1 - \sigma_3)\,n_1^2 + \sigma_3 \qquad (10)$$

$$(T')^2 = (\sigma_1 - \sigma_3)^2\,n_1^2\,(1 - n_1^2) \qquad (11)$$

Suppose now that in the latter instance F has a maximum value for a certain value of n_1. We note however that $N - N' = (\sigma_2 - \sigma_3)\,(1 - n_1^2) > 0$ by hypothesis; also $T^2 - T'^2 = n_1^2\,(1 - n_1^2)\,[(\sigma_1 - \sigma_2)^2 - (\sigma_1 - \sigma_3)^2] > 0$ since $\sigma_1 < \sigma_3 < \sigma_2$.

Therefore the same value of n_1 for $n_3 = 0$ would give rise to a greater value of F (because of hypotheses 2). Similarly $n_1 = 0$ cannot give an absolute maximum of F.

Thus we have a theorem which we may state as follows: For an isotropic medium failing under the action of three unequal principal stresses there are at every point two conjugate surface elements of probable failure which are parallel to the intermediate principal stress and equally inclined to the direction of maximum pressure at angles ϕ of less than 45°, where

$$\tan 2\phi = 1/m = F_T/F_N \qquad (12)$$

This theorem is very similar to that of Mohr; the main difference, however, lies in the statement, that the angle ϕ is determined, not only by the properties of the material, but also by the limiting stresses. Hence ϕ may vary for different stress states, a fact which allows us to describe tension fracture and shear fracture at once.

Now $m = F_N/F_T$ may be described as the relative importance of the normal stress and shear stress in producing the failure. This will allow us to divide the whole range of failure into two more or less natural divisions as follows: (a) shear failure region, wherein the shear stress is assumed to be of greater importance than the normal stress and therefore $m < 1$ and $\tan 2\phi > 1$, that is, $45° > \phi > 22.5°$; (b) tension failure region, wherein the normal tension is of greater importance, that is, $m > 1$ and $\tan 2\phi < 1$ or $0° < \phi < 22.5°$.

Thus the theory predicts that the angle of inclination of π_F to the direction of the maximum principal pressure may vary from 45° in what may be called extreme shear failure to 0° in extreme tension failure. Moreover ϕ is independent of the intermediate principal stress.

If failure occurs for some pair of values (σ_1, σ_2) then we must have $F(N, T) = 1$ for the surface element across which it occurs. Suppose we change σ_1 by a small amount $\Delta\sigma_1$ and find that a change $\Delta\sigma_2$ in σ_2 is necessary to cause failure again in an identical specimen of the same material, then we have $\Delta F = 0$ since $F = 1$ again. That is $\Delta T \cdot F_T + \Delta N \cdot F_N = 0$. Assuming little variation in n_1 over the ranges N to $N + \Delta N$, and T to $T + \Delta T$, we find from (10) that $\Delta N = (\Delta\sigma_1 - \Delta\sigma_2) n_1^2 + \Delta\sigma_2$, and from the positive root of (11) that $\Delta T = (\Delta\sigma_2 - \Delta\sigma_1)^{+}\sqrt{n_1^2 (1 - n_1)^2}$. The equation $\Delta T \cdot F_T + \Delta N \cdot F_N = 0$ becomes $\Delta T + m\Delta N = 0$. Substituting the values of ΔN and ΔT immediately above, and that of n_1^2 in (8), we find

$$\Delta\sigma_2/\Delta\sigma_1 = [(1 + m^2)^{1/2} + m]^{-2} > 0 \quad\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

calculating everything to the first order in small increments. We obtain the result that the sign of $\Delta(\sigma_1 - \sigma_2)^2$ is opposite to that of $\Delta\sigma_1$. As we go towards the shear failure region m approaches zero and this difference decreases monotonically in absolute value.

Therefore we see that the present theory predicts that at failure the absolute value of the difference between maximum and minimum principal stresses shall be least in the case of extreme tension failure, and shall increase monotonically, although at a decreasing rate, as we pass across the whole range to extreme shear failure.

To plot the relation between σ_1 and σ_2 at failure we pass to the limit of small increments in (10), obtaining

$$d\sigma_2/d\sigma_1 = [m + (1 + m^2)^{1/2}]^{-2} \quad\ldots\ldots\ldots\ldots\ldots\ldots \quad (12')$$

We also make use of the qualitative data at our command, namely

(1) $\sigma_2 - \sigma_1 > 0$

(2) For σ_2 large positively and for moderate values of σ_1 we have $m > 1$, $d\sigma_2/d\sigma_1 \rightarrow 0$ (tension failure region)

(3) For σ_1 large and negative (shear failure region) and for moderate values of σ_2 we have $m < 1$, $d\sigma_2/d\sigma_1 \rightarrow 1$

The graph of the relation would thus have the general characteristics of the curve in Figure 1.

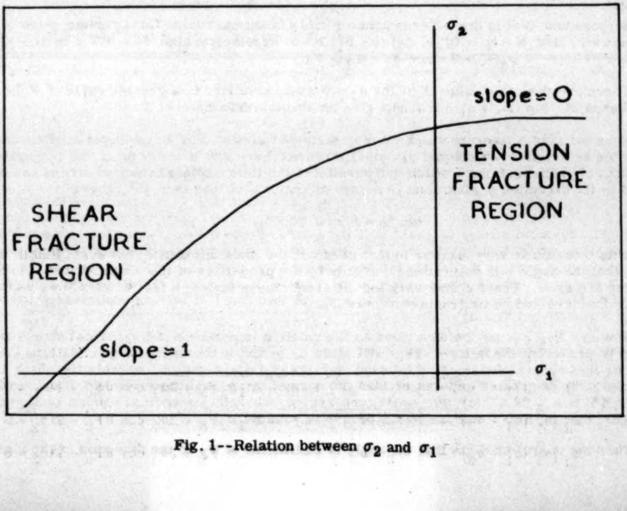

Fig. 1--Relation between σ_2 and σ_1

If this curve be obtained experimentally then the plot of ϕ as a function of the limiting stresses may be obtained from the slope values making use of (12′).

Degeneracy: two principal stresses are equal--If two of the principal stresses are equal, that is, $\sigma_2 = \sigma_3 > \sigma_1$, then the conditions for a maximum of F are no longer inconsistent. They become

$$n_1\,[(n_3^2 + n_2^2)\,(\sigma_1 - \sigma_2)^2\,F_T/T + 2\sigma_1 F_N - 2\lambda] = 0 \quad (3')$$

$$n_2\,[n_1^2\,(\sigma_1 - \sigma_2)^2\,F_T/T + 2\sigma_2 F_N - 2\lambda] = 0 \quad (4')$$

$$n_3\,[n_1^2\,(\sigma_1 - \sigma_2)^2\,F_T/T + 2\sigma_2 F_N - 2\lambda] = 0 \quad (5')$$

We note, however, that we cannot solve them for n_2 and n_3 individually but merely for $n_2^2 + n_3^2$. We find, as before that

$$n_1 = \pm \left\{[1 - m/(1 + m^2)^{1/2}]/2\right\}^{1/2} \quad (8')$$

In this instance fracture may occur equally well across any surface element tangent to the right circular cone of semi-angle

$$\phi = [\tan^{-1}(1/m)]/2 \quad (9')$$

and with axis of symmetry the direction of the maximum principal pressure. We would thus be led to expect an irregular, conical type of fracture on test specimens wherein pressure or tension is applied longitudinally, and the sides of the specimen have no applied stress. Plane fractures would be best experienced if σ_1, σ_2, and σ_3 are all distinct and differ markedly from another.

The same arguments about the variation of ϕ with the limiting stresses hold as in the non-degenerate case. If we should have $\sigma_1 = \sigma_2 < \sigma_3$, then we get instead of the above equation as maximizing condition

$$n_1\,[n_3^2\,(\sigma_1 - \sigma_3)^2\,F_T/T + 2\sigma_1 F_N - 2\lambda] = 0 \quad (3'')$$

$$n_2\,[n_3^2\,(\sigma_1 - \sigma_3)^2\,F_T/T + 2\sigma_1 F_N - 2\lambda] = 0 \quad (4'')$$

$$n_3\,[(n_2^2 + n_1^2)\,(\sigma_1 - \sigma_3)^2\,F_T/T + 2\sigma_3 F_N - 2\lambda] = 0. \quad (5'')$$

and, assuming that none of n_1, n_2, n_3 vanishes, we can solve now only for $n_1^2 + n_2^2$ and n_3^2. We find

$$n_3^2 = [1 + m/(1 + m^2)^{1/2}]/2, \quad n_1^2 + n_2^2 = [1 - m/(1 + m^2)^{1/2}]/2$$

Hence under these conditions fracture can ocuur equally well across any surface element tangent to the right circular cone with axis of symmetry the direction of minimum pressure and with semi-angle ϕ where

$$\tan 2\phi = 1/m$$

and once again an irregular conical type of fracture is to be expected.

Extension of the theory to an anisotropic medium--Our highly phenomenological approach to the facts of failure permits an extension to an anisotropic medium. Suppose that there is one plane of weakness π_ω with normal $(\alpha_1, \alpha_2, \alpha_3)$. By that we mean if we consider the value of F for a given plane we would find, for the same N and T that F would be greater for a surface element which made only a small angle than for one making a greater angle with π_ω. In other words, if we designate by $U = [\alpha_1 n_1 + \alpha_2 n_2 + \alpha_3 n_3]^2$ the square of the cosine of the angle θ which a surface element with normal (n_1, n_2, n_3) makes with π_ω we see that F is now an explicit function of U as well as N and T, and that $\partial F/\partial_u = F_u > 0$.

Now our conditions of maximum F become

$$n_1\,[n_2^2\,(\sigma_1 - \sigma_2)^2 + n_3^2\,(\sigma_1 - \sigma_3)^2]\,F_T/T + 2n_1\sigma_1 F_N + 2\alpha_1 \cos\theta F_u - 2\lambda n_1 = 0 \quad (13)$$

$$n_2\,[n_1^2\,(\sigma_1 - \sigma_2)^2 + n_3^2\,(\sigma_2 - \sigma_3)^2]\,F_T/T + 2n_2\sigma_2 F_N + 2\alpha_2 \cos\theta F_u - 2\lambda n_2 = 0 \quad (14)$$

$$n_3\,[n_1^2\,(\sigma_1 - \sigma_3)^2 + n_3^2\,(\sigma_2 - \sigma_3)^2]\,F_T/T + 2n_3\sigma_3 F_N + 2\alpha_3 \cos\theta F_u - 2\lambda n_3 = 0 \quad (15)$$

These equations define a number of surface elements for which F has relative maxima. For general values of $(\alpha_1, \alpha_2, \alpha_3)$ we note that pairs of these planes are no longer symmetrical about their respective axis since we see that if (n_1, n_2, n_3) is one solution then it no longer follows that $(-n_1, n_2, n_3)$ is also a solution.

The general solution of these equations is difficult to obtain and interpret. If, however, two of the principal stresses are equal, or if one is much greater in absolute value than the other two, as is often the case, then we may derive an interesting result. These were the conditions that introduced a degeneracy of solutions in isotropy. Let σ_2, σ_3 be the equal (or smaller) of the principal stresses, and set $\Delta^2 = (\sigma_1 - \sigma_2)^2 \approx (\sigma_1 - \sigma_3)^2 >> (\sigma_2 - \sigma_3)^2$. The last expression on the right will now be neglected with respect to the others. To this approximation our equations become

$$(1 - n_1^2)\Delta^2 \; F_T/T + 2\sigma_1 F_N + 2 \cos \theta F_u \alpha_1/n_1 - 2\lambda = 0 \; \ldots\ldots\ldots\ldots\ldots \; (16)$$

$$n_1^2\Delta^2 \; F_T/T + 2\sigma_2 F_N + 2 \cos \theta F_u \alpha_2/n_2 - 2\lambda = 0 \; \ldots\ldots\ldots\ldots\ldots \; (17)$$

$$n_1^2\Delta^2 \; F_T/T + 2\sigma_2 F_N + 2 \cos \theta F_u \alpha_3/n_3 - 2\lambda = 0 \; \ldots\ldots\ldots\ldots\ldots \; (18)$$

which gives us

$$[\alpha_2/n_2 - \alpha_3/n_3] \cos \; \theta F_u = 0$$

Since $F_u \neq 0$, and assuming for the moment that $\cos \theta \neq 0$, then we must conclude that

$$\alpha_2/n_2 - \alpha_3/n_3 = 0 \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (19)$$

that is, the plane of probable failure and the plane of weakness intersect on a line perpendicular to the direction of the distinct principal stress. The last equation removes our previous degeneracy by singling out one of the multiplicity of tangent surface elements to the cone of semi-angle ϕ. We find that n_1 is given by $n_1^2 = 1/2 + T[m/\Delta + V/\Delta^2]$, where $V = F_u/F_T[\cos \theta(\alpha_1/n_1 - \alpha_2/n_2)]$. We may assume that $\cos \theta > 0$. Also since $\alpha_2/n_2 = \alpha_3/n_3$ the sign of $\alpha_1/n_1 - \alpha_2/n_2$ is that of $\alpha_1 - n_1$. Thus the sign of TV/Δ^2 is that of $\alpha_1 - n_1$. Therefore if $\alpha_1/n_1 < 1$, n_1 is decreased, whereas if $\alpha_1/n_1 > 1$, n_1 is increased from its value in an isotropic medium. Thus, in both instances π_F is bent towards π_ω as was intuitively to be expected, by an amount roughly proportional to F_u.

There remains but one case to be considered. Is there a solution consistent with $\cos \theta = 0$, and if so what is its interpretation? If we set $\cos \theta = 0$ in (13), (14), and (15), we are back at (3), (4), and (5) of the isotropic considerations and to the dual solution of two equally likely conjugate surface elements of failure. In order that these solutions shall be consistent with $\cos \theta = 0$ we must have both these conjugate surface elements orthogonal to π_ω. That is, if $(\alpha_1, \alpha_2, \alpha_3)$ are the direction cosines of π_ω, and $(n_1, 0, n_3)$, $(-n_1, 0, n_3)$ those of the conjugate then $\alpha_1 n_1 + \alpha_3 n_3 = 0$, and also $-\alpha_1 n_1 + \alpha_3 n_3 = 0$, and since n_1 and n_3 by hypothesis do not vanish, then $\alpha_1 = \alpha_3 = 0$, $\alpha_2 = 1$, and π_ω is perpendicular to the intermediate axis of stress.

Thus failure will occur unaffected by the presence of π_ω only when π_ω is the plane of the major and minor principal stresses, and then the directions of the conjugate π_F will be exactly as in the isotropic case.

Geological significance of the isotropic case--At each point in a homogeneous isotropic medium under stress there will be two conjugate optimum surface elements of failure, their orientations depending on the stresses acting at the respective points. If we have an irregular stress distribution producing fracture then we would consequently expect an irregular type of fracture. If, on the other hand, we have a uniform stress distribution throughout the medium then all the conjugate surface elements will be parallel, and a conjugate set of plane parallel fractures is to be expected. Whether we get a single break in a medium or a multiplicity of parallel breaks would seem to depend on certain properties of the medium, particularly on the abundance of weak points at which failure might first tend to occur.

In shear zones and in rock jointing we have geological examples of a multiplicity of fractures. The latter provides probably the best large-scale examples we have of conjugate planes of fracture. Very large single fractures are in evidence in the transcurrent faults of the Canadian Shield some of which are at least 150 mi long.

The linearity of such features over great distances and through many different rock types may be attributed to a combination of circumstances. We must assume that the forces producing these fractures were uniform in character over the region and that either our function m depends but little on constants of the medium, or else these features are merely surface indications of extensive linear weaknesses in the presumably more homogeneous intermediate layer. Perhaps both are true to some extent.

Generally it is assumed that in regions other than those with extremely mountainous topography the direction of one of the principal stresses is vertical. On this assumption we can account for the following classes of faults in isotropic rock: (a) low angle thrusts, occurring when the maximum principal pressure is horizontal, the intermediate horizontal, and the minimum vertical, characterized by irregular strike and regular dip of magnitude between 22.5°-45°; (b) normal faults, occurring when the major principal pressure is vertical, and the intermediate, horizontal, characterized by irregular strike and regular dip of between 45° and 67.5°; (c) tear or transcurrent faults, occurring when the major and minor principal pressures are horizontal and the intermediate vertical, characterized by regular strike and irregular but steep dip; (d) tension cracks are predominantly very steeply dipping with more or less irregular strike since the only regional tensions occurring are likely to be horizontal.

Some simple consequences of an increase in "hydrostatic" pressure, such as would be caused by an increase in depth below the ground can be determined as follows: Suppose each stress component is decreased by the same amount, say p. This would decrease each N by an amount p as well but leave each T unchanged. By Hypothesis 2 this would decrease the probability of failure at depth. If $p = p(h)$ only where h = depth, then $\partial F/\partial h = (\partial F/\partial N)(\partial N/\partial p)(dp/dh) = -F_N\, dp/dh$.

As h increases the probability of normal failure becomes very small, that is, $F_N \rightarrow 0$, dp/dh presumably remaining finite. Thus when a depth is reached below which only extreme shear failure is possible said failure becomes independent of further increase in hydrostatic pressure.

Therefore some consequences of increase in depth are (1) all fractures would tend to die out; (2) low angle thrusts would tend to steepen to 45°; and (3) conjugate faults would tend to become orthogonal.

Significance of anisotropy--(a) Removal of degeneracy: The isotropic theory predicts not one set of parallel faults in a region that has failed under stress but two sets, one conjugate to the other. Sometimes this is actually observed in the field, but more often it is not. A possible explanation for the occurrence of only one of the pair may lie in the anisotropy of most rocks.

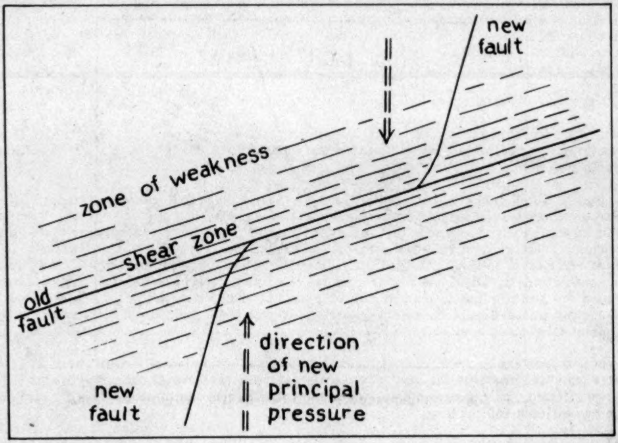

Fig. 2--Relation of an old fault to a new fault

(b) Explanation of recurrent failure along old faults: Even under different stress distributions such as may exist at different geological periods failure and resultant movement would tend to occur along the old planes of weakness. (c) Effect of an earlier fault or shear zone on the formation of another: Let us assume that we have a fault in a given locality with a certain dip and strike. The rock to either side of the fault is, as a rule, more or less sheared, the visible shearing decreasing as we go away from the fault until we enter a region where the rock has been weakened along planes parallel to the fault, but not broken or visibly sheared. The total width of this sheared and weakened rock on either side of the fault itself may vary from only a few feet to several thousand feet, depending on the amount of movement which has taken place along the fault, the magnitude of the causative stresses, and other factors. Suppose that the directions of the principal stresses have altered so that failure has now a tendency to occur along some other plane. In the unaltered rock away from the fault failure will indeed occur along this plane, but in the zone of increasing weakness the plane of weakness starts to influence the faulting and the new plane of faulting curves more and more inward towards the old.

At the old fault plane itself subsequent failure occurs preferentially along the old break, and the new fault will seem to have died out on reaching the old. If the zone of weakness is narrow there may be little apparent curvature to the new fault before it dies out. It may thus cease abruptly or appear to jog along the old fault plane to where a parallel break of the same system enters from the other side of the latter (see Fig. 2). In a similar manner an earlier shear zone may appear to displace a later fault, although it is not as likely to terminate the latter so much as would an earlier fault (see Fig. 3). The theory also predicts that in the neighborhood of the old fault plane, or in the most strongly sheared section of the zone the dip of the new fault should approximate to that of the old fault or shearing. Along such a curved fault surface there would be little movement and hence little displacement of the old shear zone.

On observing a fault to be displaced laterally along another fault or shear zone a geologist might be prone to call the displaced fault the older of the two structures, but on the basis of the above theory he would be justified in examining other evidence before drawing any conclusion as to relative ages.

Acknowledgments--The writer desires to acknowledge gratefully his indebtedness to A. E. Scheidegger who reviewed much of the pertinent literature to establish the status of this theory relative thereto, to the Ontario Research Council, and above all to J. Tuzo Wilson, without whose unceasing interest this paper would not have been written.

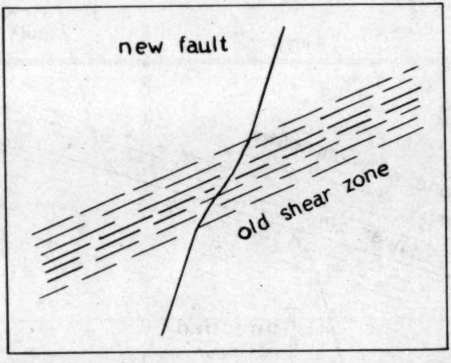

Fig. 3--Apparent displacement of a new fault by an old shear zone

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DISCUSSION OF "CLIMATOLOGY SINCE THE YEAR 1800"

by J. Leighly

[Trans., v. 30, pp. 658-672, 1949]

Leo Alpert (Intelligence Division, General Staff, Department of the Army, Washington 25, D. C.)--The writer would like to call attention to two more recent compilations of monthly mean cloudiness over the entire Earth which should be added to those listed under cloudiness.

BROOKS [1927] prepared monthly maps of mean cloudiness over the entire Earth (actually 90° N to 80° S) from which he computed values of mean cloudiness over the oceans and land masses by ten-degree belts around the Earth. Although the original maps were not published, copies of the January and July maps may have been included in SIMPSON'S [1929] classical paper, "The distribution of terrestrial radiation." Simpson wrote: "The surface of the Earth between latitude 70° N and 70° S was divided into 10° squares, for each of which Brooks determined the mean temperature and the mean cloud amount for each of the two months, January and July. The values are shown on Figures 2, 3, 4, and 5." Brooks' values of the mean monthly and annual cloudiness over the Earth derived from the original maps were published in the Memoirs of the Royal Meteorological Society [BROOKS, 1927]. These values have been used extensively in computations of the albedo of the planet Earth, the distribution of terrestrial radiation, and the terrestrial heat balance. It is interesting to note that the maps which Leighly stated were apparently the latest compilations of mean cloudiness over the Earth (actually 80° N to the coast of Antarctica at approximately 70° S) and published in Shaw's Manual of Meteorology [SHAW, 1928; ed. 2, 1936], also were compiled by Brooks in 1919 and revised by him in 1921.

The writer believes that the latest monthly compilation of mean cloudiness over the entire Earth (actually 80° N to the coast of Antarctica at approximately 70° S) was prepared by H. Landsberg in 1944. His monthly maps were based on data for approximately 8000 continental and island stations, maps prepared by the U. S. WEATHER BUREAU [1938], a publication by BROOKS and CONNOR [1936], and numerous other sources. Landsberg's original maps, on a scale of approximately 1:30,000,000, have apparently been lost. However, smaller scale and less detailed copies for January, March, May, July, September, and November have been published [LANDSBERG, 1945]. Though parts of the analysis are open to criticism, without doubt these are the most reliable maps of mean cloud distribution over the entire Earth. Of course, no maps of this sort can be considered reliable until the defect of insufficient information over a 24-hr period is overcome.

In addition to these monthly values, two maps (1:80,000,000) showing the state of the sky over the Earth from 84° N to 59° S (for January and July only) have been published [GREAT SOVIET WORLD ATLAS, 1937]. The following legend is used: clear, cloudiness slight for more than half of the month, as much as 0.2 of the sky; overcast, cloudiness great for most of the month, more than 0.8 of the sky; half clear, slight, medium, and great cloudiness is observed to an equal extent (about 1/3 of the month each), or cloudiness for more than half of the month is from 0.3 to 0.7 of the sky.

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DISCUSSION OF "FLOOD FREQUENCIES AND SEDIMENTATION FROM FOREST WATERSHEDS" by HENRY W. ANDERSON

[Trans., v. 30, pp. 567-586, 1949]

Henry W. Anderson (Author's closure)--The history of flood studies might be said to consist of seeking answers to: "How big can floods get?", then "How often will floods come?", and now "What causes floods?" - or design flood studies, flood frequency studies, and flood cause studies. Hydrologists have lately devoted more and more time to the last. Peak discharges and sedimentation from watersheds as related to storm and watershed variables are one part of these flood cause studies.

Schiff's discussion [Trans., v. 30, pp. 584-586, 1949] of our report on such a study brings out some important points. In the present primitive state of development of the relationship of peak discharges to meteorological and watershed variables such suggestions are sure to be helpful. He suggests and presents data to show that for two Ohio watersheds the effects on discharges of increasing forest cover varied considerably between two areas, one of .001 and one of .068 sq mi. In the smaller of these (his Watershed 134) he reports significant reductions in peak discharges occurred when forest cover was changed from a density of ten to 95 per cent. This was due to a reduction in surface runoff. Of the larger (his Watershed 172) he makes the statement that, "Had the actual peak discharges been used to establish an equation for Watershed 172, increases in cover would seem to have little influence. Better cover increased permeability rates and consequently infiltration rates so that most of the flow of Watershed 172 is now subsurface flow which reappears above the measuring flume. Peak discharges have not materially reduced and reflect the particular soil and geologic conditions of Watershed 172." This is an interesting interpretation, and the factors which he talks of are probably operating; however, the results on peak discharges seem to be not at all what he hypothesizes. First, we would expect a reduction in peak discharges when surface runoff is changed to subsurface flow. And second, that such a reduction occurs and is of large magnitude may be shown when Schiff's data are actually "used to establish an equation." Taking the data from his Table 1 and testing the relation of Q_{172}, the peak discharges in inches depth per hour to the associated per cent cover density (C), with the maximum hourly precipitation (P_1) in inches per hour as the meteorological control we find that

$$\text{Log } Q_{172} = 0.197 + 1.320 \log P_1 - 0.398 \log C$$

The regression coefficient for cover density is found to be highly significant and the magnitude of the effect indicated is such that associated with the increase in cover from ten to 95 per cent there was a reduction of 61 per cent in the peak discharges. This is certainly not an insignificant decrease. With more storm data it would be possible to test other meteorological parameters and improve the equation but probably the cover density effect would remain about the same, since no obvious correlation appears between cover density and any of the meteorological characteristics listed in his Table 1. The improvement in estimates of the discharges with the addition of the cover variable is shown in Figure 1 where the discharges estimated from the equation (given above which includes cover as a variable) are compared with those from an equation using only maximum hourly precipitation, P_1

$$\text{Log } Q_{172} = -0.431 + 1.169 \log P_1$$

Schiff's data show that cover effects vary with the size of the watershed although the effect for his larger watershed was not zero as he surmised. It might be well to test if any such variation can be detected for the Southern California study where the watersheds studied ranged in area from one to 202 sq mi. We test the joint variable, log cover density over log watershed area, in the equation $Q = f(A, P_{24}, aP, C, C/A)$ using the 127 storm-watershed data of our Tables 1 and 2. The partial regression coefficient for log C over log A was not significant (the t in the t-test was -0.955 whereas a value of 1.98 was required for significance at the five per cent level), neither was the correlation coefficient higher than for our Eq. (2) in the original paper. We conclude that no significant variation of cover effects with watershed size has been demonstrated for our data by what seems like

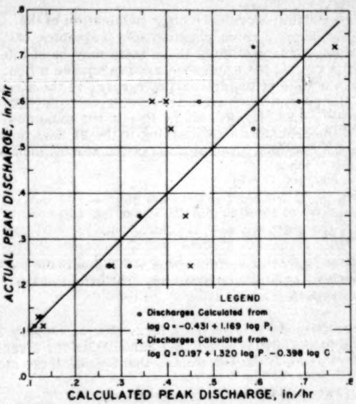

Fig. 1--Comparison of actual discharges, Watershed 172, with discharges calculated from (1) an equation using maximum one-hour precipitation; and (2) an equation which included forest cover density as well as maximum one-hour precipitation as variables

the appropriate test. However, Schiff's findings and interpretation of the mechanism of discharge reduction indicate that tests should always be made.

Schiff suggests that watersheds be segregated by size groups before analysis. There may be some advantages to this, but as a general procedure there are also some disadvantages. First, areal effect on discharge is a continuous function - no natural discontinuity exists. Second, by the restricting of the data to areal groups it is possible that we may eliminate just that diversity in other variables which would permit evaluation of their effects. It would seem better when possible to include areas of variable sizes, evaluate the areal effects, and rely on joint tests such as made above to test for any distortions or curvilinearity in the functions used.

Schiff remarked on the close agreement between the frequencies obtained from the actual discharges for a 30-yr period and the discharges calculated from the storm data for the same period (see Fig. 3, original paper). This came about despite considerable dispersion of the individual values about the trend (see Fig. 2, original paper). The dispersion, of course, arises from the fact that a single rain gage gives a poor estimate of watershed precipitation for some storms. The good frequency determination arises because of the large number of peaks and storms which were used, including many unpublished peaks which were read from the original U. S. Geological Survey hydrographs. As for the mechanics of drawing the trend which Schiff inquired about, an easy way to get a close approximation the first time is to rank the actual and the computed discharges in order of magnitude, plot the ordered pairs, and draw the line through these. The deviation of the trend from equality gives the correction to be applied to the calculated discharges. After correction, the computed discharges should give about the same frequency as the actuals.

Perhaps the most important question is whether the error in the determination of frequency is likely to be less for the method developed than for other methods which have been used in the past. Without introducing direct comparisons, it can be said that several new elements have been introduced which were shown to be highly important. Chief of these was the difference in discharges measured under variable watershed cover conditions - correction factors as high as 8.8 were involved in bringing some discharges to a comparable basis so they could be used in frequency determinations. A method of extending the period of discharge records to try to obtain better representation of time variation was developed and used. The extension of the time period by a minimum of 39 yr would be expected to have improved the estimate of the frequencies of peak discharges

Schiff's warning that results from one area must be used with caution in another area is well taken. We can go further and say that for each area with varying conditions, an equation must be tested before being used. We have had occasion to make some such tests for watersheds of the Santa Ynez and Santa Maria drainages in California. The chief differences between these watersheds and those of the original study are that in the new watersheds sedimentary rocks are dominant, instead of igneous and metamorphic rocks. Although differences in the constants were found, the functions were remarkably similar, those for the discharge function in (3), the precipitation function of (4), and the cover functions in (2) and (4) being nearly identical. However, as a general rule, if sufficient data are available it is better to develop new equations including parameters designed to permit expression of all the important physical effects on discharges.

Schiff's thoughtful discussion is appreciated. His suggestions that many more analyses are needed, and that flexibility of approach is desirable, are seconded. We might emphasize that, in applied hydrology, such flexibility must be exercised within rather strict requirements as to time and space representativeness.

Physiography as a watershed variable--An opportunity developed after publication of the original paper to make further tests of the effect of some stream physiography variables of HORTON [1945] on peak discharges from watersheds. The results of these tests may be of interest: where Q is the peak discharge from a watershed in cfs; A, the watershed area in square miles; C, the average cover density on the watershed at the time of the discharge; $r_b r_s / r_1$ is the stream bifurcation, slope, and length ratios and PE_3 is the precipitation effectiveness as calculated from the equation $\log PE_3 = 1.188 \log P_{24} + 0.516 \log aP + 0.985 \log P_1$, where P_{24} is the maximum 24-hr precipitation during a storm, inches; aP is the precipitation occurring in the 21 days antecedent to the storm, inches; and P_1 is the maximum one-hour precipitation of the storm, inches; we obtained from the original data listed in Tables 1 and 2.

$$\text{Log } Q = 1.3383 + 1.060 \log A - 0.975 \log C + 0.863 \log PE_3 + 0.360$$

$$(\log PE_3 \times \log r_b r_s / r_1) - 0.477 \log r_b r_s / r_1$$

The correlation coefficient was 0.954 and the standard error of estimate was ± 0.273 with the partial correlation coefficient for the stream physiography being highly significant. The effect of the stream physiography is thus seen to vary with storm effectiveness.

By way of illustration, (see Table 1) if we compare the effects for a large storm, January 23, 1943, with a smaller storm, January 8, 1940, using San Gabriel 2 and Big Santa Anita for a range in physiography (all data given in Tables 1 and 2 of original paper), we see that for the large storm San Gabriel 2 would be expected to have a 57 per cent higher unit discharge than Big Santa Anita and for the smaller storm a 21 per cent higher discharge due to stream physiographic difference alone. Comparisons of the actual difference show 62 and 56 per cent higher unit discharges of San Gabriel 2 for these particular storms. Because of the difficulty of obtaining watershed storm precipitation accurately, deviations in individual estimated amounts are often high for a single storm on a single watershed, but for all storms and watersheds the results obtained by regression should be about right. The results suggest that the stream physiography parameters developed by R. E. Horton may be useful in estimating discharge-producing potential of watersheds. The recent increased interest shown by many hydrologists in flood-cause studies gives promise of rapid development of criteria and methods for improved water and watershed management.

Table 1--Comparison of a large and a small storm on two watersheds

Watershed	1943 storm			1940 storm		
	Log PE_3	Log $r_b r_s / r_1$	Effect[a]	Log PE_3	Log $r_b r_s / r_1$	Effect[a]
San Gabrial 2	3.07	0.62	3.040	1.75	0.62	1.603
Santa Anita	3.06	0.33	2.846	1.71	0.33	1.519
Difference			0.194 = 57 per cent			0.084 = 21 per cent

[a] Effect = $0.863 \log PE_3 + 0.360 (\log PE_3 \times \log r_b r_s / r_1) - 0.477 \log r_b r_s / r_1$.

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NOTES AND PERSONALIA

NOTES

International exchange of persons under the Fulbright Act--The State Department announces the availability of more than 1000 opportunities for Americans to undertake graduate study, teaching, or research abroad during the academic year 1951-1952. Similar opportunities are available for a like number of foreign nationals to come to the United States. Grants to Americans usually include transportation, tuition or stipend, and living allowance, including books and equipment. Grants to foreign nationals coming to the United States include round-trip transportation only. Expenses in the United States must be met from other sources. Those interested in collegiate teaching or research grants should address the Conference Board of Associated Research Councils, Committee on International Exchange of Persons, 2101 Constitution Avenue, Washington 25, D. C. Those interested in graduate study should address the Institute of International Education, 2 West 45th Street, New York 19, N. Y. Closing date for applications is October 15, 1950.

Shell Oil Company fellowships--The Shell Oil Company is continuing its graduate fellowship program through the academic year 1950-1951, and has announced the 31 colleges and universities in which the 45 fellowships have been placed. The colleges and universities make the selection of the fellows. A number of the fellowships are in geophysics, while others are in closely allied fields. Persons desiring information on the program should address Shell Fellowship Committee, 50 West 50th Street, New York 20, N. Y.

Survey of the Gulf Stream--Between June 6 and June 23, 1950, six research vessels from the United States and Canada, manned with 350 men, including 50 scientists, made an intensive investigation of the Gulf Stream. The main purpose of the survey, named Operation CABOT was to gather data on the meandering of the Stream between Cape Hatteras and the tail of the Grand Banks of Newfoundland. Cooperating in the work were the U. S. Navy Hydrographic Office, the Canadian Naval Research Establishment, the U. S. Fish and Wildlife Service, Woods Hole Oceanographic Institution, and Scripps Institution of Oceanography. Richard H. Fleming was the senior scientist on the work; William L. Ford was the chief Canadian scientist; V. B. Colton was the chief scientist of the Wildlife Service; Martin J. Pollak was in charge of the Atlantis; and Bostwick H. Ketchum, of the Caryn.

Symposium on the Theory of Electromagnetic Waves--A Symposium on the Theory of Electromagnetic Waves, sponsored jointly by New York University and the Geophysical Research Directorate of the Air Force Cambridge Research Laboratories, was held at New York University June 6-8, 1950. The program included 21 scientific and technical papers. More than 200 attended, representing 24 universities, 18 industrial organizations, 20 government laboratories, and three foreign countries. It is anticipated that a bound volume of the papers will be issued in the spring of 1951.

New geochemical journal--A new international journal, Geochemica et Cosmochimica Acta, is to be published by Butterworth-Springer, London, under the editorship of L. R. Wager. Papers will be published in English, French, and German with summaries in English. American authors should send their communications to Earl Ingerson, U. S. Geological Survey, Washington 25, D. C., who is the American editor. Subscribers in the United States should communicate with the Academic Press, Inc., 125 East 23rd Street, New York 10, N. Y.

Danish Deep-Sea Expedition--The Danish Government research vessel Galathea is scheduled to depart from Copenhagen in October, 1950, on a two-year round-the-world cruise via the Cape of Good Hope and the Panama Canal. She will conduct researches in marine biology, terrestrial magnetism, and hydrography. The Galathea, formerly the British frigate Leith, has been renamed in honor of the corvette of that name which carried the first Danish Deep-Sea Expedition in 1845-1847.

Centenary Supplement to the Quarterly Journal of Meteorology--A supplement of 200 to 250 pages, covering the papers and discussions of the Centenary Symposia in Oxford and London in the spring of 1950, will be published by the Royal Meteorological Society. Copies ($3.00) may be ordered by addressing the office of the Society at 49, Cromwell Road, London, S. W. 7, England.

British National Institute of Oceanography--The work of the former Discovery Committee, formerly under the Colonial Office, has been resumed by the National Institute of Oceanography, under the British Admiralty. The William Scoresby has been recommissioned and sent to the South Atlantic to investigate whales and oceanography in South African and Australian waters. The Discovery II left London late in April 1950 for work in the Antarctic. She has been equipped to obtain long cores from the ocean bottom and to do seismic reflection studies of ocean sediments. The work will be under the direction of H. F. P. Herdman, with J. F. Blackburn in executive command.

New Spanish publication in science--The Pan American Union has recently issued Number 1 of its new Boletín de Ciencia y Tecnología, the purpose of which is primarily to bring the results of scientific and technological work in the United States to the attention of Latin-American scientists. It will also carry news of scientific matters likely to be of special interest in the Latin-American countries.

Changes in the Department of Geology at Columbia University--At Columbia University the Department of Geology, continuing its policy of rotating administrative duties, has elected Walter H. Bucher to serve as Executive Officer for a period of three years. He succeeds Paul F. Kerr whose term expired on June 30, 1950, and who held office during the difficult period of post-war reconstruction. Marshall Kay has been elected Educational Coordinator, and Paul F. Kerr, Research Coordinator. These three officers constitute an Executive Committee for the Department.

New edition of data on altitudes in the United States--The U. S. Geological Survey has recently issued a revised edition of "Extreme and Mean Altitudes in the United States, its Territories and Possessions." It also includes similar information on the continents. This consists of four pages, and may be secured through the Survey, Washington 25, D. C.

Bibliography of interest to geophysicists--A Bibliography of Organic Reefs, Bioherms, and Biostromes, edited by W. E. Pugh, has been published by the Seismograph Service Corporation, Tulsa, Oklahoma. This contains over a thousand entries, chiefly bearing on geology, but many of interest to geophysicists also.

Geophysical seminars of the Air Force Cambridge Research Laboratories--The Friday afternoon seminars of the Geophysical Research Directorate, Air Force Cambridge Research Laboratories are continuing during the summer months. The ones on August 25 and September 1 will be conducted by Joseph Kaplan of the University of California on the subject "The Airglow" based on light of the night sky. They are currently being held in Room 3-270, Massachusetts Institute of Technology.

Report on the supply of and demand for geologists--The Committee on Geological Personnel of the American Geological Institute has recently issued its annual report, indicating that the number of students in geology is now at an all time high, and that the demand outlook is such that many trained in this field may not find employment, professionally, in geology. Copies of the report may be obtained in writing the Institute at 2101 Constitution Avenue, Washington 25, D. C.

Connecticut Agricultural Experiment Station to celebrate its Diamond Anniversary--The Connecticut Agricultural Experiment Station, oldest in the United States, will celebrate its seventy-fifth birthday by ceremonies on September 28 and 29, 1950. The opening day will be devoted to appropriate general festivities, including an open house and two addresses of general interest. The second day will be a formal Symposium on the Research Institute in Modern Society. Particular homage will be paid to Samuel W. Johnson, founder of the Experiment Station.

Publications of Centre de Recherches et d'Etudes Oceanographiques (CREO)--Five types of publications are issued by the Centre. The first two, (1) technical information and bibliographies (known as R. T. B.), and (2) technical instructions (known as I. T.), are issued as circulars, about twelve annually. The other three are (3) publications or pamphlets containing original work on oceanography including submarine geology, of which about six will be issued annually, (4) minutes of open meetings relating to applications of oceanography, and (5) annual reports of CREO. The first two circulars, covering respectively a bibliography of current meters, and dealers the world over of oceanographic supplies, have been issued. Other early ones will deal with granulometric studies, and a bibliography of the salinity determinations. The first of type (3) under the authorship of M. L. Berthois will relate to lithologic studies of underwater sediments. The circulars covering types (1) and (2) are available on one subscription, and the other publications as a second. Prospective subscribers should write to the Centre at 1, Quai Branly, Paris VII^e, France.

PERSONALIA

P. L. Bediz is now working with the Century Geophysical Corporation as Supervisor in their Canadian operations.

Joseph C. Boyce, until recently Chairman of the Department of Physics, College of Engineering, New York University, has accepted a position as Associate Director of Argonne National Laboratory in Chicago.

William R. Chalker, formerly on the research staff of the Department of Meteorology, Massachusetts Institute of Technology, is now engaged as a meteorological consultant with A. H. Glenn and Associates in New Orleans, dealing especially with the problems of the forecast of ocean swell and wind velocity as they relate to oil drilling operations in the Gulf of Mexico.

Kazim Ergin has received the degree of Doctor of Philosophy from California Institute of Technology and will soon return to his former position with Maden Tetkik ve Arama Enstitüsü in Ankara, Turkey.

Frank A. Horton, recent graduate in Civil Engineering of Northwestern University, has accepted a position as civil engineer in hydrologic research with the Tennessee Valley Authority.

Ray K. Linsley, Head of the Hydrologic Branch and Assistant Chief, Climatological and Hydrologic Services of the U. S. Weather Bureau, has resigned to take up duties as Associate Professor of Hydraulic Engineering at Stanford University.

Franklin C. Raney has accepted the Pack Forest Soils Assistantship in the Department of Agronomy of Cornell University where he is pursuing studies toward a doctorate.

E. Raymond Shepard, Geophysicist with the Corps of Engineers, Department of the Army, recently retired after 28 years of service.

V. P. Sokoloff has resigned his position with the U. S. Geological Survey and is engaged in teaching and research work in soils in the Isaiah Bowman School of Geography of Johns Hopkins University.

Athelstan F. Spilhaus, Dean of the Institute of Technology, University of Minnesota, has been named Deputy Chairman of the Committee on Geophysics and Geography of the Research and Development Board, succeeding William W. Rubey.

Harlan T. Stetson has retired as Director of the Cosmic Terrestrial Research Laboratory at Needham, Massachusetts, and will make his home at Fort Lauderdale, Florida.

Lorenz G. Straub, Director of St. Anthony Falls Hydraulic Laboratory and Head of the Civil Engineering Department at the University of Minnesota, was recently awarded the French Medal of Academic Palms.

Pao-Chang Paul Sun is now with the Scientific Department of the Ingalls Shipbuilding Corp. at Pascagoula, Mississippi, on stress analysis work.

Warren O. Wagner has established an office as a Consulting Hydraulic Engineer in Pasadena, California. Previous to this, he had devoted eight years to teaching and other work with California Institute of Technology.

Charles M. Weiss, formerly Chemist-Biologist with the Baltimore Harbor Project of Johns Hopkins University, has accepted a position as Biologist with the Water Pollution Control Division of the Public Health Service with headquarters at 42 Broadway, New York.

Jacob B. Wickham, Associate in Oceanography, Scripps Institution of Oceanography, is engaged on a cooperative program of Scripps Institution, Woods Hole Oceanographic Institution, U. S. Naval Electronic Laboratory, and the U. S. Hydrographic Office on investigations of time changes in sea ice.

Norman R. Beers, Editor of Nucleonics, died June 27, 1950 in New York at the age of 38. He was born at Trimble, Missouri, received his baccalaureate degree in 1933, and the degree of Master of Arts in 1934, both from the University of Missouri. He also received a baccalaureate degree from Oxford University in 1937. Before the War, he served in the Bureau of Aeronautics of the Navy Department. He joined the Navy in 1941 and was assigned duty on the faculty of the Postgraduate School of the Naval Academy, where he remained in a civilian capacity as Associate Professor of Aerological Engineering until 1947, when he became associated with Brookhaven National Laboratory in charge of the Meteorology Group. In 1949, he took up duties as editor with the new McGraw-Hill publication, Nucleonics. He had been a member of AGU since 1945.

Albert K. Ludy, retired, formerly Chief of the Section of Geomagnetism of the U. S. Coast and Geodetic Survey, died in Denver, Colorado on June 10, 1950, at the age of 68. He was born in Philadelphia, and received his baccalaureate degree from the University of Pennsylvania in 1903. He retired from the Survey in 1944 after 25 years of service, including periods in Alaska, Arizona, and Maryland before becoming Chief of the Section in Washington, D. C. in 1942. He had been a member of AGU since 1930.

H. Shaw, Director of the Science Museum in London, died May 4, 1950, at the age of 58. He was born at Huddersfield, Yorkshire, England. He attended London University, securing his baccalaureate degree in 1913; the degree of Master of Science in 1922, and the degree of Doctor of Science in 1931. In 1920, he became Assistant Keeper of the Department of Physics and Geophysics of the Museum, advancing to Keeper in 1935, which post he held until be became Director of the Museum in 1945. He had been a member of AGU since 1947.

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MINUTES OF EXECUTIVE COMMITTEE MEETING, AMERICAN GEOPHYSICAL UNION

Meeting of May 3, 1950

The meeting was called to order at 19^h30^m in the Board Room of the National Research Council - National Academy of Sciences Building, 2101 Constitution Avenue, Washington, D. C. by President Walter H. Bucher. The newly elected officers, though they do not take office until July 1, 1950, were invited to attend.

Present--Thirty-six persons were present, namely: L. H. Adams, L. R. Alldredge, K. Hilding Beij, Merrill Bernard, Francis Birch, Walter H. Bucher, W. S. Burbank, Horace R. Byers, Felix Chayes, Ernst Cloos, Herbert N. Eaton, Jno. A. Fleming, Richard H. Fleming, R. C. Gibbs, H. W. Hemple, Albert J. Hoskinson, M. King Hubbert, C. E. Jacob, W. C. Jacobs, H. Freeborn Johnston, J. Wallace Joyce, David G. Knapp, W. D. Lambert, Esper S. Larsen, Jr., Ray K. Linsley, J. Putnam Marble, L. M. Murphy, Frank Neumann, A. C. Redfield, Florence Robertson, Irwin Roman, James B. Small, McClain B. Smith, Waldo E. Smith, Harlan T. Stetson, and George P. Woollard. The following were excused: Fred Anderegg, Robert Balk, J. Bjerknes, J. E. Church, L. O. Colbert, Ralph E. Cleland, Richard E. Fuller, James B. Macelwane, M. A. Mason, W. A. Noyes, Jr., C. O'D. Iselin, F. W. Reichelderfer, Roger Revelle, V. C. Stechschulte, W. D. Sutcliffe, and James T. Wilson.

Approval of Minutes--The minutes of the meeting of October 28, 1949 were approved as published in the December, 1949 issue of the Transactions, pages 928 to 939.

General Secretary's Report

This Committee, as you doubtless are well aware, is not only the Executive Committee of the American Geophysical Union but also is the Committee on Geophysics of the National Research Council and, as such, is the American National Committee of the International Union of Geodesy and Geophysics. This unique interrelationship is of inestimable worth to the American Geophysical Union. It seems appropriate, therefore, to present this report under two main headings corresponding to the two main functions of the Committee.

General Activities

Committee Membership--We have suffered the loss by death of Harry D. Harradon, longtime member of this Committee and Chairman of the Committee on Statutes and By-laws. McClain B. Smith succeeds him. Arthur Bevan, whose term as Chairman of the Division of Geology and Geography of the National Research Council expired June 30, 1949 is succeeded by Ernst Cloos.

Office Personnel--Mrs. Sherman F. Fortner, Jr., clerk-typist, resigned February 17, 1950, and her place has been taken by Miss Hilda A. Borgos.

Finances--A detailed financial report as of June 30, 1950, will be submitted to the Committee on Audit. As of March 31, 1950, our balance was $20,171.54, including investments of $3008.00 at cost. This compares with a balance of $18,421.65, including investments of $2,768 at cost, as of March 31, 1949. The increase of $1,649.89 is the direct result of the increase in dues taking effect this year.

Membership--The Membership Committee has secured 225 new members since our last meeting. Resignations have been somewhat larger than usual because of the increase in dues so that the net gain has been 22 members, as shown below:

Membership (corrected), October 26, 1949	4,573	
New members, October 27, 1949 to April 28, 1950	225	4,798

Resignations	175	
Deceased	17	
Inactive	3	
Dropped	8	203
Membership, April 20, 1950		4,595

Seventeen deaths have been reported since our last meeting, namely Henry F. Moore, January 8, 1948; Joseph A. Cushman, April 16, 1949; Stephen R. Capps, January 19, 1949; Wm F. Prouty, June 27, 1949; Robert E. Fellows, September 19, 1949; Raymond W. Pierce, September 22, 1949; Nephi W. Cummings, October 31, 1949; Wm. J. Humphreys, November 10, 1949; Frank B. Jewett, November 17, 1949; Harry D. Harradon, December 8, 1949; Chas. J. Clifford, December 21, 1949; C N. Fenner, December 24, 1949; Isaiah Bowman, January 6, 1950; I. P. Tolmachoff, January 17, 1950; Curt H. Eiffert, February 3, 1950; Richard C. Farrow, February 20, 1950; and Alonzo H. Tuttle, March 5, 1950. I also wish to mention Benno Basch (not a member), engraver of the dies for the Bowie Medal, who died April 21, 1950.

As required by the By-Laws, I report that 114 members have failed to pay dues for two years (1948 and 1949), and recommend that they be dropped from our rolls. A further recommendation is made, that, in accordance with past custom, the General Secretary be authorized to make one more effort to bring these members into good standing before they are dropped.

We now have 42 life members, eight through payment of life dues of $100, and 34 because of retirement according to provisions of the Statutes.

Corporation Memberships--Since our last meeting three new corporation members have been added making a total of seven.

Administration--Arrangements have been made with the Carnegie Institution of Washington to continue in our present offices at least through January 1951.

Ballots for officers of the Union and its Sections for the triennium beginning July 1, 1950, were sent out late in February and returns were received until April 4. The President appointed the following Committee of Tellers after consultation with Section officers: Earl Ingerson (Chairman); Gunnar Leifson and E. J. Parkin (Geodesy); Benjamin L. Snavely (Seismology); Robert D. Fletcher and Philip Clapp (Meteorology); H. F. Johnston and J. R. Balsley (Terrestrial Magnetism and Electricity); J. H. Hawley, Harry S. Ladd, Boyd E. Olson, and E. O. Hulbert (Oceanography); Herbert E. Hawkes and Robert R. Coats (Volcanology); G. Earl Harbeck, Jr. and Max A. Kohler (Hydrology); L. H. Adams and Harold M. Bannerman (Tectonophysics).

On recommendation of Martin A. Mason, Chairman of the Committee for the Study of Tsunamis, the President appointed M. P. Cuellar a member of the Committee.

Ernst Cloos, one of our two directors on the board of the American Geological Institute, has resigned, and George P. Woollard has been appointed for the remainder of the term expiring November 30, 1951.

Meetings--The Twenty-Fifth Regional Meeting was held in Davis, California, February 3 and 4, 1950, with about 200 in attendance. The Sections of Hydrology and Meteorology cooperated in holding the meeting and they were joined by a small informal group from the University of California which is interested in evaporation problems.

The Thirty-First Annual Meeting, just concluded, had a registered attendance of 865, and probably about 100 others were present who did not register.

Publications--Work is progressing on the index for the first 25 years of the Transactions, and it is hoped that this can be published in four or five months.

The program and abstracts of papers for the annual meeting this year were published as Part 2 of the April, 1950 Transactions.

On February 10 and 11, 1950, the National Research Council sponsored a meeting of editors and publishers of scientific journals to discuss problems of such publication. The Executive Secretary attended both sessions and the General Secretary the second session.

Regionalization--The President has written to Section officers calling attention to the published plan [Trans. v. 30, Dec. 1949, pp. 936-939] for regionalization and inviting their use of it whenever suitable occasions arise. It is to be emphasized that the plan is wholly voluntary on the part of all Sections, and the part of the Union itself will be merely that of furnishing support when such support is requested, or calling attention of Sections to opportunities which may arise without obligating the Sections. It is noted that Nevada, shown to be in the South Rocky Mountain Region in the published plan, has been placed in the South Pacific Region instead.

Miscellaneous--On invitation of the Royal Meteorological Society the President designated C. O'D. Iselin, Woodrow C. Jacobs, H. E. Landsberg, Jerome Namias, C. S. Piggott, F. W. Reichelderfer, and Harry Wexler as representatives of the American Geophysical Union at the Centennial Meeting of the Society.

Editor Walter B. Langbein has continued to remit $70 of each of his $75 semi-annual honoraria. These remittances are transferred to the Permanent Fund as a contribution from Mr. Langbein.

Activities as Committee on Geophysics of the National Research Council

The privileges accruing from our functions under this heading are, broadly speaking, the prestige of association with the parent body and the availability of advice and assistance from this body, and the opportunity of engaging in the international aspects of geophysics in our position as the American National Committee of the International Union of Geodesy and Geophysics. Thus the National Research Council furnishes meeting places to the full extent of its resources, and annually grants a modest sum for our expenses. In the international field our members have great opportunities for expanding their scientific horizons through organized international cooperation.

Our duties under this heading, in addition to carrying out such specific tasks as the National Research Council may request, are mainly in the international field. Thus, we are sending approximately 200 copies of our Transactions gratis to officers of the International Union and its Associations, to recognized research organizations, and to government institutions, libraries, and universities. Nominally, this service is on an exchange basis. Pertinent items in the publications received are listed in the bibliographies in our Transactions and the publications are sent to the Library of the Weather Bureau, the Library of Congress, or other recognized repository.

Another item is general correspondence facilitating world-wide contacts of our members with scientists abroad.

Bibliography of Hydrology--As its part in the preparation of an international bibliography sponsored by the International Association of Hydrology, the Section of Hydrology, through a Special Committee, is considering ways and means for publishing a new issue covering hydrology in the United States for the years 1941 to 1950, inclusive. At present it appears that the difficult financial problem is near a solution.

International Union of Geodesy and Geophysics--The Ninth General Assembly will be held in Brussels from August 21 to September 1, 1951. Questions relating to Association programs should be sent to appropriate Section Secretaries. In particular, it is essential to give notice of proposed papers at once, and advisable to submit manuscripts (at least three copies) before January 1951. Questions relating to International Union affairs should be sent to the Executive Secretary. He should be advised at the earliest possible moment of intentions to attend. Persons planning to attend should initiate travel reservations at once, particularly for travel by ship.

Colloquium--An international colloquium on "Plastic Flow and Deformation within the Earth," being arranged jointly by the International Unions of Theoretical and Applied Mechanics and of Geodesy and Geophysics, will be held in Washington, September 12 to 14, 1950. L. H. Adams is Secretary of the Joint Organizing Commission.

Action on the General Secretary's report--The report was approved, including the recommendations concerning members in arrears in dues.

International matters--J. A. Fleming reported briefly on the activities of the International Council of Scientific Unions in its relations with UNESCO and IUGG. UNESCO has made limited grants for carrying on various scientific work. It does not desire to support continuing activities, only to assist on special projects, and to get permanent activities under way. A grant of $3000 has been made to the IUGG and a similar amount to the International Union of Theoretical and Applied Mechanics for traveling expenses of foreigners to the Colloquium on "Plastic Flow and

Deformation within the Earth." He mentioned the Ninth General Assembly of IUGG to be held in Brussels, Belgium, August 21 to September 1, 1950.

L. H. Adams advised that plans for the Colloquium - involving about 10 or 12 persons from abroad, and a similar group from the United States - are taking shape.

Reports of Standing Committees

Membership--Chairman Small reported that in the last year there have been about 500 new members.

Corporation Membership--Chairman Roman reported a total of seven corporation memberships. Work is under way to secure more. He recommended that the designation be changed from a sub-committee to a special committee. This was approved by vote.

Meetings--Chairman Marble presented a summary of the report shown in Appendix 1. The report was accepted.

Editorial and Publication--Chairman Beij presented a summary of the report shown in Appendix 2. The report was accepted. It was moved and passed that the Editorial and Publication Committee should consider and report on (a) the desirability of having at least one qualified reviewer of each paper in addition to the editor so that the minimum requirement for acceptance is favorable review by two qualified reviewers, including the editor, and (b) a method of notifying authors periodically of the status of submitted papers. Chairman Beij also reported briefly on the two-day meeting called by the National Research Council in February on editorial and publication problems related to scientific journals.

Budget--Chairman Eaton presented the budget for the next fiscal year as shown in Appendix 3. After extensive discussion, this was adopted.

Reports of Special Committees

Establishment of a Journal of Geophysical Research--In the absence of Chairman Reichelderfer, the General Secretary presented the report of the Committee (see Appendix 4) which was accepted, and the Committee was discharged with thanks for its careful and detailed studies [see also Transactions of December 1949, p. 931 and pp. 935-936]. It was the consensus of the meeting that this matter should be kept in mind for future action, should conditions for the publication of a second journal become favorable.

Study of Tsunamis--In the absence of Chairman Mason, the General Secretary noted that an annotated bibliography on tsunamis, containing about 200 references, had been prepared by Marcial Cuellar of the Committee. This is to be published soon by the Hydrographic Office.

Geophysical and Geological Study of Continents--Pending preparation of a written report, Chairman Woollard outlined briefly the progress made in securing field data from oil companies and others, most relating to gravity observations. These are being worked up, and it is anticipated that the basement complex map, prepared in 1939, will be revised.

Cosmic-Terrestrial Relationships--Chairman Stetson outlined the report presented as Appendix 5.

Resolutions--In the absence of Chairman Ingerson, J. Putnam Marble reported on the action taken at the Business Meeting during the afternoon of May 3, 1950 [see Transactions of June 1950, pp. 486-487].

Student Membership and Student Chapters--Chairman Jacobs presented the report shown as Appendix 6. It was moved and passed that the report be received and that the recommendations of the report be referred to the Committees on Budget, Statutes and By-Laws, and Editorial and Publication for study, and for report to the next meeting of the Executive Committee, and that the Committee which presented the report be asked to stand by for consultation.

Tellers--The report of the Committee of Tellers on the recent triennium elections was presented at the Business Meeting during the afternoon of May 3, 1950. It was moved and passed that the winners only be announced, and that the actual tally of the votes be kept on file for the confidential information of any voting member in good standing who makes a request, in writing, for the information.

Reports of Sections

Seismology--President Neumann stated that two sessions might be necessary to take care of the program for the Section next year.

Terrestrial Magnetism and Electricity--President Roman mentioned that a committee has been organized for the handling of papers and other matters relative to the Brussels Assembly in 1951.

Oceanography--Secretary R. H. Fleming announced that the Executive Committee of the Section would act as the committee for handling papers and other matters pertaining to the Brussels Assembly in 1951.

Volcanology--President Burbank announced the organization of a Section Committee on Parícutin to take over the functions and complete the work of the similar Committee in the NRC that has been terminated.

Hydrology--Secretary Linsley reported briefly on the organization of the Section to take care of matters relating to the Brussels Assembly in 1951.

Tectonophysics--President Hubbert reported that though only two weeks before the program deadline, only three papers were in evidence, the Section finally arranged two full and excellent sessions.

Reports of Divisions of the National Research Council

Physical Sciences--Chairman Gibbs outlined the activities of the Division especially as they relate to AGU, and advised that the volume "The Internal Constitution of the Earth" has been revised, and is in the hands of the publisher. He presented a report on the work of the Committee on Geophysics Advisory to the Office of Naval Research, which is being published as part of the Report of the Division of Geology and Geography.

Reports of Representatives to Affiliated Organizations

American Geological Institute--George P. Woollard, one of the Directors from AGU, presented the report shown in Appendix 7, on behalf of our two Directors.

American Standards Association Committee Z59--C. E. Jacob advised that his report has been made to the Section of Hydrology.

Unfinished and New Business

Emblem--The General Secretary noted that we have had a request from the Milwaukee Public Library for a copy of the Union's "emblem" for use with others of scientific societies in a frieze in the Science Room of the Library. An emblem is being prepared, using the obverse of the Bowie Medal as on the front cover of the *Transactions* with a modified portion of the face as used at the head of the program with the words "American Geophysical Union - Unselfish Cooperation in Research." This action was approved.

Special Committee to Study the Need for a Section of Geochemistry and Radioactivity--It was moved that a Special Committee be appointed to look into the desirability of establishing a new Section relating to geochemistry and radioactivity and report to the Executive Committee. After considerable discussion, this was approved.

Greetings to R. M. Field--It was moved and approved that Past President Richard M. Field be sent a telegram as follows: "The Executive Committee in session assembled sends you greetings, and appreciation of many past tasks that you have done so well."

Adjournment--The meeting adjourned at 22^h22^m.

K. HILDING BEIJ, General Secretary
WALDO E. SMITH, Executive Secretary

APPENDIX 1--REPORT OF THE COMMITTEE ON MEETINGS

John Putnam Marble, Chairman

J. A. Duerksen O. W. Torreson Frank J. Veihmeyer
Nicholas A. Rose Merle A. Tuve George G. West

During the past year, the Twenty-Fifth Regional Meeting was held at the University of California at Davis by the Sections of Hydrology and Meteorology, with an attendance of upwards of 200.

The Thirty-First Annual Meeting was held in Washington, D. C. on May 1-3, 1950, in the rooms of the building of the National Academy of Sciences - National Research Council and the Auditorium of the General Services Building. The registered attendance was 865, the largest on record. In connection with the scientific sessions, which were held by all Sections, there were the following Union functions: on the evening of May 1, the Presidential Address of Walter H. Bucher, followed by the Twelfth Award of the William Bowie Medal to Leason H. Adams, and a social hour with light refreshments; on the evening of May 2, an informal smoker in the Auditorium of the Cosmos Club, attended by about 180 members and their guests; and on the afternoon of May 3, a general session of the Union, at which motion pictures of scientific interest were shown, followed by an address by David M. Delo, Executive Secretary of the American Geological Institute, outlining the programs and aims of the Institute, and its relations to the Union, which is one of the Founder Societies of the Institute, and by a brief business session of the Union as a whole.

It has been announced that the Western Snow Conference will meet in Victoria, B. C. in April, 1951, and an invitation has been extended to the AGU to participate. This has been referred to the Section of Hydrology, which is taking appropriate action.

In accordance with the authority extended by the Executive Committee, the Committee on Meetings, upon consultation with the appropriate officers, has set the dates April 30-May 2, 1951 (Monday-Wednesday) as the dates for the Thirty-Second Annual Meeting of the Union, in Washington, D. C. At the moment it is planned to use the same meeting rooms as this year, but this is subject to change. Further announcements will appear in the Transactions from time to time.

APPENDIX 2--REPORT ON PROCEDURES ON ACCEPTANCE AND REJECTION OF PAPERS FOR THE TRANSACTIONS

In accordance with a motion passed at the meeting of the Executive Committee on October 28, 1949 [see Transactions, December 1949, p. 932], I submit herewith a summary report of the procedures in accepting papers and reports for publication in the Transactions and in recommending rejection for action by the Editorial and Publication Committee as a whole. Procedures vary somewhat according to the needs of the various Sections. However, the following generalizations can be made:

(1) All Editors accept papers on their own responsibility when they feel qualified to pass judgment, except the two Editors for Hydrology. By formal action of long standing, papers in Hydrology are always referred to chairmen of the appropriate research committees of the Section for initial review.

(2) When Editors feel they are not qualified, or if they feel doubts as to merits of papers in their own field, they send the manuscripts to one or more independent reviewers.

(3) Only "crackpot" and unquestionably worthless papers are commended for rejection without independent review.

(4) By action of the Editorial and Publication Committee, each paper recommended for rejection by an Editor is considered at a meeting of the Committee and rejected by majority vote. Papers may be rejected for several reasons. Some good papers, generally because of their being in the engineering field, are rejected with the recommendation that they be submitted to more appropriate journals. "Crackpot" and obviously worthless papers are flatly rejected. In many cases the rejections are in the form of recommendations to re-study the subject with a view to meeting the objections raised by the reviewers.

The detailed statements of the Editors are on file in the office of the Union.

K. Hilding Beij, Chairman
Editorial and Publication Committee

APPENDIX 3--REPORT OF THE COMMITTEE ON BUDGET FOR THE FISCAL YEAR ENDING JUNE 30, 1951

Herbert N. Eaton, Chairman

Jno. A. Fleming N. C. Grover

The Committee on Budget of the American Geophysical Union has conferred with the General and Executive Secretaries of the American Geophysical Union regarding the budget for the fiscal year, July 1, 1950, to June 30, 1951, and proposes the following bedget. As the Committee predicted last year, the increase in dues from $5 to $7 has placed the Union on a self-sustaining basis.

Statement of Assets and Liabilities as of June 30, 1950 (estimated)

Assets:			
General Funds:			
General cash reserve	$10,500		
Reserve for membership list	500		
Reserve for travel of Executive Secretary to international meetings	250	$11,250	
Pernament Fund:			
U. S. Gov't. bonds (at cost)	$ 2,368		
Northwest Federal Savings and Loan Account (Deposits only)	640	$ 3,008	
Robert E. Horton Fund		1,059	
William E. Bowie Medal Fund		100	$15,417
Bills receivable			2,800
			$18,217
Liabilities:			
Bills payable		$ 300	
Surplus		17,917	$18,217

Operating Statement (General Funds)

Estimated receipts:			
National Research Council, grant		$ 400	
Reserved last year for preparation of membership list		500	
Income:			
Dues, 4,250 members	$29,750		
Dues, 7 corporation members	700		
Sales, *Transactions*	5,000		
Sales, reprints	2,000	37,450	
Total receipts			$38,350
Estimated expenditures:			
Publication of *Transactions*:			
Printing	$ 8,000		
Preparation	7,800		
Co-editors	1,500		
Mailing and postage	1,000	$18,300	
Reprints		2,000	
General office expenses:			
Salaries	$11,900		
Stationery and miscellaneous	1,200		
Postage and communication	1,200		
Equipment	400	$14,700	

Expenses, meetings:			
Annual	$ 600		
Regional	300	$ 900	
Expenses, Sections		200	
Expenses, Committees		600	
Travel of officers		100	
Reserve for travel of Executive Secretary to international meetings		250	
Membership list		1,000	
Total expenditures			$38,050
Estimated excess of receipts over expenditures			300

Operating Statement (Robert E. Horton Fund)

Estimated receipts	$ 0		
Estimated expenditures	1,059		
Excess of expenditures over receipts			$ 1,059

Operating Statement (William Bowie Medal Fund)

Estimated receipts		$ 0	
Estimated expenditures		20	
Excess of expenditures over receipts			$ 20

Statement of Assets as of June 30, 1951 (estimated)

General Funds:			
General cash reserve	$10,800		
Reserve for travel of Executive Secretary to international meetings	500		$11,300
Robert E. Horton Fund			0
William Bowie Medal Fund			80
Permanent Fund, investments			3,008
Bills receivable less bills payable			2,500
Total assets[a]			$16,888

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[a] Does not include value of office equipment or of surplus stock of Transactions.

APPENDIX 4--REPORT OF THE SPECIAL COMMITTEE ON PROPOSAL FOR A NEW JOURNAL OF GEOPHYSICAL RESEARCH

F. W. Reichelderfer, Chairman

L. H. Adams
H. N. Eaton
E. A. Eckhardt
H. W. Hemple

In a meeting of the Executive Committee on April 23, 1949, Merle A. Tuve presented a proposal for a new journal in the general field of geophysics. After lengthy discussion the Executive Committee adopted a motion requesting the President to refer the proposal for thorough study by the Editorial and Publication Committee or by a special committee appointed for this purpose. In his letter of June 13, 1949, President Bucher appointed a special committee to "gather necessary information from the outside and report to the Executive Committee at the October meeting if possible." A Preliminary Report was submitted to the Executive Committee by the Special Committee under date of October 28, 1949 [Trans. Dec. 1949, pp. 935-936]. The principal issues were briefly summarized and attention was invited to the need for further study. Members of the Special Committee assisted by the Chairman of the Editorial and Publication Committee and the

Executive Secretary of the American Geophysical Union have subsequently gathered considerable information aimed at answering the following general questions.

(1) What would be the advantages and disadvantages of a new journal of geophysical research as a means for advancement of the science?

(2) What would be the cost of publication?

(3) How could the Journal be financed?

The Special Committee obtained views from members of the Executive Committee and other members who had expressed interest and had views as to the desirability of a new journal. The experience of other scientific organizations was reviewed with reference to cost of publication and questions incident to planning and financing a new publication. The principal sources of information entering into the findings of the Special Committee are listed in the last page of this report. Before final recommendations had been reached, information was received from Tuve that his further study of costs and discussion with the Executive Secretary had led to the conclusion that his proposal should be reconsidered and accordingly on February 16, 1950 he wrote as follows:

> "I have made a careful study, with Waldo Smith, of the expected changes in costs if my proposal for publishing at least half of the present AGU Transactions by letter press is adopted. This will run to at least $5000 per year extra cost. I have also examined the question of incorporating additional material, by consolidation of other journals. The inclusion of material which we now publish in the Journal of Geophysical Research, for example, would run roughly $8000 additional per year, and would not bring in anywhere near this amount of additional income.
>
> "Estimates by Waldo Smith run rather higher than our estimates, but this is in the correct direction, as an increase in the amount submitted for publication must be expected.
>
> "I do not visualize sources for such greatly increased income for the AGU. Therefore I withdraw the proposal I made a year ago for changing our AGU publication arrangements."

The action of the Executive Committee which led to appointment of the Special Committee applied specifically to Tuve's proposition. With withdrawal of the proposal the question before the Special Committee is presumably settled.

With reference to the first question relative to advantages and disadvantages of the new Journal as a means for advancing the science of geophysics there appears to be general agreement with President Bucher's view that the journal would encourage publication of worthwhile papers in this field and increase the opportunities of the AGU to show leadership in advancement of the science. Inquiries by the Special Committee as to the volume of worthwhile papers brought information that there should be no lack of material to publish although there is not a large backlog of good technical papers awaiting publication at this time. Compared with other fields of science, publication delays in geophysics are not serious. Some members of the AGU invited attention of the Committee to the probable effects of a new journal on other journals now published in this field and one or two suggested the desirability of considering a broader merger of publications. Most of these views have been brought to the attention of the Executive Committee in previous discussions and are therefore not presented in detail here. Majority opinion appears to favor a new journal if a sound plan can be found to finance it.

Estimates of cost of publication range from approximately $15,000 additional per year to $25,000 or $30,000 per year. Inquiry into the experience and costs of other scientific organizations leads to the conclusion that the additional cost per year for publishing the Transactions and a new journal of the format and scope discussed before the Executive Committee would be at least $20,000.

Consultation with the Executive Secretary, the General Secretary, and others familiar with the financial circumstances of the AGU brought the conclusion that the new Journal could not be financed except by a relatively large increase in dues or by contributions from some extraordinary source. The Committee was unable to bring out any definite prospects for another source of funds for this purpose.

In order to facilitate further consideration of the subject if desired by the Executive Committee, either now or at some future time, the correspondence of the Special Committee and notes with reference to conferences on costs, and other material, have been retained in file. A list of the principal papers follows:

(1) Letter from M. A. Tuve dated April 26, 1949, proposing a new Journal of Geophysical Research.

(2) Background comments by K. Hilding Beij, Waldo E. Smith, and Walter H. Bucher, accompanying letter of June 11, 1949 by President Bucher.

(3) Letter of June 13, 1949 by President Bucher, appointing Special Committee.

(4) Letters from Presidents or representatives of Sections of the Union and from other members of the Union with reference to desirability of a new journal (June through November 1949).

(5) Copies of letters from American Society of Mechanical Engineers (September 24, 1949), American Physical Society (October 3, 1949), and American Meteorological Society (May 13, 1948 and September 24, 1948), with reference to publication costs of their journals. Summary of National Research Council meeting with reference to publication problems of various scientific journals (four pages forwarded in Waldo E. Smith's letter of February 14, 1950, and other informal notes with reference to publication costs).

(6) Letters from Tuve dated February 15 and February 16, 1950, and from President Bucher dated March 1, 1950, relative to financial quiestions and Tuve's withdrawal of the proposal.

APPENDIX 5--REPORT OF THE SPECIAL COMMITTEE ON COSMIC-TERRESTRIAL RELATIONSHIPS FOR THE YEAR 1949-1950

Harlan T. Stetson, Chairman

C. F. Brooks
J. H. Dellinger
J. A. Fleming

S. A. Korff
D. H. Menzel

G. W. Pickard
J. Q. Stewart
W. T. Thom, Jr.

The Committee on Cosmic-Terrestrial Relations, formed in 1938, completed its twelfth year of activities on June 30,1950. During this period, the interrelation of many geophysical fields has been studied particularly as concerns the work of two other special committees, one, the Special Committee on the Geophysical and Geological Study of Oceanic Basins and the other, the Special Committee on the Geophysical and Geological Study of Continents.

The problem of possible variations in geographic coordinates, so much concerned with crustal movements and tectonics, received the early attention of this Committee. Attempts at re-evaluations of some conflicting evidence have been made that concern the question of the reliability of determining lateral shifts, considering the order of precision now available in geodetic observations.

The important question of interpretations of apparent variations in longitude is of great interest to both these other special committees. The validity of any inference involves the question of the order of accuracy obtainable in time determinations transmitted across continental and oceanic areas. The last decade has seen improvements in instrumentation of time keeping mechanisms. The latest and most promising chronometer that may ultimately prove more reliable than the rotation of the Earth itself is the atomic clock whose possibilities are yet to be explored and realized.

Whatever the precision obtainable in time comparisons, the usefulness in the application of any instrument in determining variations in longitude must depend upon the degree of precision with which time, once determined, can be transmitted between two meridians. Since time signals are transmitted by radio waves whose transmission paths depend upon atmospheric and ionospheric conditions, this Committee has given particular emphasis to promoting investigations of radio-wave propagation.

A tendency for a progressively increasing value in longitude differences observed during longitude campaigns has been noted, and the trend mentioned suggests a correlation with the progress of the season of the year. Since intensive longitude campaigns have advisedly occurred during the autumn of the year because of clearer skies for star observations at that time of the year, a seasonal effect could not be eliminated by repeated observations unless equally and sufficiently satisfactory data were accumulated the year around. Such a trend as has been noted causing systematic increases in longitudes with the advent of winter may yet be found to be attributable to some vagaries of radio wave propagation not yet fully allowed for. This question, therefore, remains open to further study.

Another equally important unanswered question concerns the fundamental postulate that radio waves used in transmitting time signals are propagated in the direction of the east-west vector with the same speed as when the direction is that of the west-east vector. This is a fundamental postulate in the determination of all longitudes and remains yet to be established with the precision to be desired. As a result of rapid advances in technique and instrumentation following the war, some progress has been made towards the ultimate test of the validity of this hypothesis. The answer as to the soundness of this fundamental premise is to be hoped for through the cooperation of those now engaged in the field of radio wave propagation at various frequencies.

It is still to be regretted that the long 17-kc wave transmission in time signals across the Atlantic is not continuously maintained in this country. This has been emphasized recently to the Chairman of your Committee in conference by the Royal Astronomer of Great Britain. It may be recalled in an earlier report of this Committee that prior to the outbreak of war in 1941, arrangements had been made to re-establish the 17-kc radio time signals from Washington. This, however, was not actually accomplished before war indefinitely postponed such transmissions under the stress of the national emergency. It is these very long waves, of approximately 17,000 m in length that can be counted upon for reliable interchange of time signals across the Atlantic, summer and winter, since they are least subject to disturbances coincident with geomagnetic storms and solar activity. It is to be emphasized that the problem is still of sufficient basic geophysical significance to be of special interest to the two special committees cited and that this problem remains over as one not yet satisfactorily solved. As has been summarized in the previous reports of this Committee, the work done in the last few years has emphasized the subject of radio-wave propagation as influenced by cosmic factors. An important contribution to this field has been the maintaining of continuous recordings of transmitted signals over fixed distances at fixed frequencies. In this, there has been close cooperation between the various members of the Committee particularly those representing the Central Radio Propagation Laboratory of the National Bureau of Standards and its cooperating stations.

Since 1941, the Cosmic Terrestrial Research Laboratory at Needham, Massachusetts has maintained continuous standardized field intensity records of the standard frequency signal broadcast from WWV 5 Mc over a distance of 373 mi. Similar records of field intensities received over the same distance from WWV 10 Mc transmitter have been maintained since 1945 and of fields from the 15 Mc transmitter since 1946. These recordings, together with those of other cooperating laboratories, have afforded a large amount of material for analysis over a sufficient period of time to establish with considerable confidence certain relationships between field intensity patterns and sunspot numbers.

Comparisons of the records at Needham with predicted conditions for transmission have yielded satisfactory confirmation of anticipated ionospheric transmission. There remain many problems, concerning transmission far from solved, such as transmissions of high-frequency waves by way of reflections from the E-layer otherwise known as sporadic E. This problem is being attacked by competent experts in the field. It is gratifying to note the increasing number of those qualified who contribute to the whole general field of radio-wave propagation which has been greatly stimulated under the stress of National emergency. It is to be pointed out that in addition to the well-recognized solar effects, lunar effects upon the ionosphere have become fairly well established irrespective of agreement as to the mechanisms involved.

Meanwhile, work of the committee has recognized important relationships between radio-wave transmission at very high frequencies and atmospheric conditions which has stressed the interrelation of the science of communications and meteorology. The series of measurements of field intensities on the 44.1 Mc and 92.1 Mc bands transmitted from Alpine, New Jersey and recorded beyond the horizon at Needham at a distance of 167 mi has been previously reported. Marked relationships have been shown to exist between atmospheric refraction at the Earth's surface and the passing of warm and cold fronts. These series of measurements were terminated at Needham in December 1948 for 44.1 Mc and for the 92.1 Mc frequency band in June 1949. Numerous papers dealing with results of these investigations have appeared and some are in process of publication. With the completion of ten years' recording of the Bureau of Standards standard frequency signal, field intensity measurements of WWV 5 Mc and 15 Mc will terminate at Needham June 30, 1950. The laboratory at Needham will be closed as of that date and certain other types of Cosmic Terrestrial Research will be transferred to Fort Lauderdale, Florida, and will be continued in that location.

Altogether at the close of this series, approximately 151,680 hr of continuous recordings of WWV on the three frequencies will have been obtained.

Certain active members and collaborators of the Committee who have been most cooperative have been recently retired from their respective institutions. These include J. A. Fleming, Director of the Department of Terrestrial Magnetism, Carnegie Institution of Washington, J. F. Hellweg, Superintendent, Naval Observatory, John H. Dellinger, Chief, Central Radio Propagation Laboratory, National Bureau of Standards, and your Chairman so far as his association with the Massachusetts Institute of Technology is concerned.

During the past year D. H. Menzel has continued his studies relative to a new theory of solar activity in relation to solar terrestrial relationships. Intensive studies have been given to polar spicules. Menzel finds that the areas of greatest spicule activity are in the mysterious "M-regions" associated with geomagnetic disturbances. Matter shot upwards and partially supported by the Sun's magnetic fields appears to comprise much of the corona. If the density of the gas is great enough, the material cascades to solar surfaces carrying with the lines of magnetic force, the focusing action resulting in the formation of prominences and on occasion even of sunspots. The spots arising appear to occur almost entirely in the solar atmosphere and are not believed to extend deeply into its surface. Menzel believes that terrestrial effects may arise from such spicules or portion of spicules that reach the Earth. He conjectures that spicule matter near sunspots release large amounts of ultra violet light resulting in "radio noise" at frequencies greater than 30 Mc.

The possible relation of cosmic-ray intensity to solar phenomena has been receiving renewed emphasis. S. E. Forbush, Thomas Stinchcomb, and Marcel Schein have called the attention of your Committee of four sudden increases in cosmic ray intensity associated with solar flares or chromospheric eruptions that have been observed during a decade of continuous registration of cosmic ray intensity. The most recent and largest sudden increase occurred on November 19, 1949, recorded at the high altitude observing station at Climax, Colorado. In this instance, the intensity increased 200 per cent above normal within a half hour. At Cheltenham, Maryland, near sea level, the increase was only about 43 per cent. No increase was observed at the equator.

It is concluded that the increase in the effect with altitude and latitude indicates that the effect was due to the nucleonic components produced by relatively low-energy, highly-charged particles probably accelerated by some solar mechanism.

S. A. Korff has continued active in this field of cosmic-ray investigations and in the improvement of Geiger counters as recorders. An excellent summary by Korff on "The genesis of cosmic radiation" has been published in the January, 1950 issue of Physics Today. Korff critically examines what we know of cosmic rays and on various hypotheses that have been advanced as to their origin such as from the Sun, the galaxy, intergalactic space or, as Lemaitre suggests, a possible fossil radiation left over from nuclear catastrophic action occurring with the initial explosion in which, according to the expanding universe theory, our universe was formed. A significant contribution "Cosmic radiation and the maintenance of a potential gradient in the atmosphere" by Korff appeared in Physical Review v. 77, pp 144, 1950.

In the past year G. W. Pickard has continued his studies and analyses of the long records of WBBM field intensity measurements, the last 12 years of which have been made at the Cosmic Terrestrial Research Laboratory in Needham. To connect with the series of measurements begun by Pickard in 1926 in the Boston area and the series carried on in Cambridge and in Lexington, it has been necessary to gain information in all changes in antenna heights, powers and frequencies transmitted from the Chicago station. This information has been obtained through the cooperation of the Columbia Broadcasting Company and the Federal Communications Commission and the searching of the files of correspondence that has taken place during the interval.

Accordingly, it has been possible to tabulate on a reasonably satisfactory basis field intensities of WBBM in terms of microvolts per meter at one mile corrected for all changes in power, antennas at the transmitter and with reasonable assurance, the various effective heights of the antenna at the receiving end. It is possible that errors of ± 20 per cent may occur in attempting to reduce to absolute values the entire series. Of course, relative changes in field intensities on the basis of which various correlations of field intensities have been made are of considerably higher accuracy. Since, however, field intensities encountered in practice throughout the day and season vary by several hundred per cent, the new results now tabulated should be of considerable value in the study of long-term and periodic trends.

With all of the data available, Pickard has established new correlations with geomagnetic disturbances from 1926 to 1929. It is anticipated that the results of some of these analyses will be published shortly.

From the meteorological field, C. F. Brooks reports as follows on a joint project of the U. S. Weather Bureau and Blue Hill Observatory:

"The solar work at the Blue Hill Observatory continued to include total insolation on a horizontal surface (record started in 1932), insolation received at the bottom of a tube ten times as deep as its diameter kept at normal incidence (since 1932), diffuse insolation from the sky on a horizontal surface (since 1945), total insolation on a vertical surface facing south (since 1945). A record of the insolation on a vertical surface facing north was begun in July 1949, as well as experiments with a new, waterproof tube for the solar beam at normal incidence connected to a Brown Electronic Potentiometer and General Electric Integrator. A R. W. Cramer Co. Running Time Meter was attached to the Micromax Recording Potentiometer connected to the old solar tube, to count the hours and minutes when the intensity of the solar beam at normal incidence exceeded 0.3 cal cm^{-2} min^{-1}, which is approximately the intensity required to activate the Campbell-Stokes sunshine duration recorder.

"Two papers were published: 'Atmospheric contamination over Boston, Massachusetts' by I. F. Hand, with discussion by C. F. Brooks, in the *Bulletin* of the American Meteorological Society, September 1949, v. 30, pp. 251-254, and 'Insolation on clear days at the times of the solstices and equinoxes for latitude 42°N,' by I. F. Hand, in *Heating and Ventilating*, January 1950.

"Irving F. Hand, Meteorologist in charge of the Weather Bureau's Solar Radiation Field Testing Unit at Blue Hill, set off in March 1950 on a seven-months' trip for the purpose of checking the solar stations of the United States, Mexico, and Canada. To the 30-odd older stations in the network, more than 50 new ones have been added in the last two or three years, each with a 10-junction Eppley Pyrheliometer and a Brown Electronic Potentiometer. Some stations record on strip charts, others with integrators.

"Routine observations of the aurora (66th year) continue. The record cannot be considered as homogeneous with respect to auroras that are confined to low arches in the north, owing to the great increase in city illumintion in that direction since the record began."

APPENDIX 6--REPORT OF THE SPECIAL COMMITTEE ON STUDENT MEMBERSHIPS

Woodrow C. Jacobs, Chairman
Florence Robertson

At a meeting on December 3, 1948, the Executive Committee of the American Geophysical Union authorized the President to appoint a special committee to consider the question of student memberships in the Union; to determine the feasibility of a student membership, and, if feasible, to develop a tentative plan for consideration by the Executive Committee.

The Committee met upon the occasion of the meetings in Washington, D. C., the spring of 1949. At that time the Committee came to certain conclusions but withheld its report until such time as a more thorough survey of university conditions could be made and a representative cross section of the AGU membership could be consulted. The original conclusions drawn by the Committee remain essentially unchanged as a result of the more thorough survey.

The Committee has failed to discover among the universities and other institutions of higher learning any significant student population that could be conveniently and economically brought into the Union under a "student group" or "chapter" from of active membership. It has become apparent to the Committee that a fair fraction of the graduate students in the several fields of geophysics can be economically contacted by the faculty individuals who are members of the Union and these students can usually qualify for at least the associate grade of membership.

A larger number of such students could be induced to join the Union if the dues for student members (or associate members) could be lowered to more nearly meet the student budget. Annual dues of not more than $3.00 are usual for student memberships in other scientific and technical societies. The Executive Secretary of the Union estimates that the cost of publishing an additional copy of the *Transactions* is of the order of $2.50 per year. It is obvious, therefore, that a significant student membership would not be a source of revenue to the Union.

Nevertheless, it would appear that the Union should do everything possible to foster and stimulate teaching and research in geophysics within the universities and to attempt to attract, at whatever cost, the better student minds to one or more of the several sciences which comprise the field of geophysics.

The student membership should apply only during the time the member in actively enrolled as a student (which might perhaps be extended to include the first year following cessation of active enrollment). It would be hoped that a large fraction of the student membership would continue as full members of the Union upon the termination of the student classification. Of necessity, the number of student members would be small and perhaps the number of members added to the AGU would be insignificant compared to the total membership. This small segment of membership, however, might eventually be counted among the more active forces within the Union.

The following recommendations are offered:

(1) The Committee recommends the establishment of a student grade of member with the annual dues not to exceed $3.00.

(2) The student member shall be granted all the rights and privileges of full members of the AGU except the right to vote.

(3) The classification of student member shall terminate after one year of non-student status. The termination of student status shall be automatic and, if judged to be qualified by the Executive Committee of the Union, the former student member will be considered an associate member of the Union. The written acceptance of this procedure shall be considered one of the requirements of student membership.

APPENDIX 7--REPORT TO THE AMERICAN GEOPHYSICAL UNION BY ITS REPRESENTATIVES ON THE BOARD OF DIRECTORS OF THE AMERICAN GEOLOGICAL INSTITUTE

W. T. Thom, Jr. George P. Woollard

American Geological Institute activities were initiated approximately a year ago with the appointment of David M. Delo as Executive Director. Since that time the Council of the Institute has held two meetings and its Executive Committee has held several additional meetings. The work of the Institute to date has been largely exploratory and development, in order that its program be developed constructively and in ways that will best serve the joint interests of its constituent societies. The "Report to the Geological Profession" issued by the Institute gives a summary of its activities; charts showing its plan of organization and its relation to the Division of Geology and Geography of the National Research Council, and a financial statement indicating disbursements to date and the funds available to cover these expenses and activities for the coming year. The work of the Institute to date has been centered chiefly upon consultation, distribution of information, education, and a study of the supply and demand situation within the geologic-employment field. Those desiring details of these activities should consult the News Letter of the Institute, especially No. 8.

SPECIAL ANNOUNCEMENTS

New directory of members to be published in October--It is planned that a new list of Members will be Part 2 of the October issue of the Transactions. Corrections of the addresses of members should be received by AGU not later than September 10, 1950 for inclusion in the new directory.

Regionalization plan now in effect--The plan for regionalization outlined in the December 1949 issue of the Transactions [pp. 931, 936-939, and 942] is now in effect. The plan is unchanged from that set forth in the Report, except that Nevada is to be a part of the South Pacific Region instead of the South Rocky Mountain Region. The degree of activity under the plan is largely up to the individual members and to the Sections. All Sections are free to regionalize or not according to the desires of members residing in the region. The main purpose of regionalization is to bring the activity of the Union closer to the members through the arrangement of coordinated regional meetings, either by AGU alone or in cooperation with other scientific groups, national or local. Those interested in the development of such a regional program should write to the office of AGU, indicating in which Sections there is desire for such regional activity.

Ninth General Assembly of the International Union of Geodesy and Geophysics--It is now just a year (August 21-September 1, 1951) until the Ninth General Assembly of IUGG will be under way in Brussels, Belgium. Those planning to go by ship should take steps at once to arrange for passage, and those going by airplane should arrange their passage early in 1951. Word of papers for the program should be forwarded to the office of AGU without further delay. Those who plan to attend should notify the office of AGU.

Thirty-Second Annual Meeting of AGU--The Thirty-Second Annual Meeting of AGU will be held in Washington, D. C., April 30 to May 2, 1951 (Monday to Wednesday). See the Special Announcements in the October issue for details with regard to program plans.

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